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Wave propagation and dispersion in anisotropic media through bond-based peridynamic theory

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ABSTRACT

Research aim: Peridynamic theory is a mathematical framework that employs integro-differential equations rather than the traditional partial differential equations that are used in classical continuum mechanics. An investigation into the plane wave has been carried out in this study in order to ascertain the dispersion relation for plane waves in an anisotropic medium. The dispersion relation for longitudinal and transverse wave is determined by analyzing the harmonic solutions. Validation was achieved by ensuring classical correspondence in the limit of the nonlocality parameters approach zero. The general solution to the initial-value problem is found, and the closed-form formula for displacement in terms of Green's function is derived for the very first time for anisotropic materials. In this study, nonlocality functions such as Gaussian, linear, and constant functions are investigated. The variations in frequency, phase velocity, and group velocity that occur as a result under the influence of the size of the horizon (which reflects nonlocal length) and nonlocality functions, graphs are presented and obtained findings are discussed.

Literature survey: The peridynamic framework is well-suited for evaluating fracture and failure in anisotropic materials because of its inherent ability to handle discontinuities[1]. There is barely any literature on anisotropic materials in the peridynamic framework. Trageser et al.[2], [3] have made significant contributions to the field of bond-based peridynamics. In their important works, they have established a bond-based framework for anisotropic media. The models they have developed are subject to constraints imposed by Cauchy relations. The works encompassed both conventional and unconventional state-based frameworks for anisotropic materials. Scabbia et al. [4]conducted a study on the modelling of anisotropic materials in both 2D and 3D using general ordinary state-based peridynamics. This modelling approach incorporates two bond stiffness functions. Weckner[5] provided an integral representation for the plane wave solution utilising Green's functions technique for isotropic material. However, the dynamic behaviour of anisotropic material within the framework of peridynamic theory has yet to be addressed in the literature. Through theoretical modelling and numerical simulations, this article aims to uncover the intricate interplay between nonlocal behaviours and wave propagation on anisotropic material through peridynamic theory.

Problem formulation:

The equation of motion of peridynamic theory with no external body force density within the domain $\mathcal{H}_{\mathrm{x}}$, a ball of radius δ centred at x, as mentioned by Trageser [2], is as follows

$$\rho(x)\ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \int_{\mathcal{H}_{\mathrm{x}}} \boldsymbol{\lambda}(\boldsymbol{\xi})\boldsymbol{\xi} \otimes \boldsymbol{\xi}\big(\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{\xi},t) - \boldsymbol{u}(\boldsymbol{x},t)\big)d\boldsymbol{\xi}\,. \quad (1)$$

The micromodulus tensor is given as

$$\boldsymbol{\lambda}(\boldsymbol{\xi}) = \frac{1}{m}\frac{(\xi\otimes\xi)L(\xi\otimes\xi)}{\|\xi\|^4}\frac{j_\delta(\|\xi\|)}{\|\xi\|^2}. \quad (2)$$

The weighted volume is mentioned below

$$m = \int_{\mathcal{H}_x} j_\delta(\| \boldsymbol{\xi} \|) \| \boldsymbol{\xi} \|^2 \, d\xi. \tag{3}$$

Where, $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$ is a position vector such that $|\boldsymbol{\xi}| = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$. L is a fourth-order symmetric tensor related to the classical elasticity tensor, defined by Trageser [2]. The nonlocality function is denoted by $j_\delta(\| \boldsymbol{\xi} \|)$, and is defined in [2]. Also, the density is denoted by ρ .

On substituting the harmonic solution $\boldsymbol{u} = \boldsymbol{a}\, e^{i(\boldsymbol{kx}-\boldsymbol{\omega}t)}$ we obtain an eigenvalue problem.

$$M\boldsymbol{a} = \lambda\, \boldsymbol{a}\,, \tag{4}$$

where $\lambda_1, \lambda_2, \lambda_3$ are eigenvalues. The dispersion relations for plane wave in anisotropic media are given as

$$\omega_i = \sqrt{\frac{\lambda_i}{\rho}},\ i = 1, 2, 3 \tag{5}$$

Significant conclusions: In conclusion, both longitudinal and transverse waves have dispersive characteristics due to the nonlocal qualities inherent in the peridynamic medium. The dispersion is affected by the nonlocal length parameter, also known as the horizon size, as well as the nonlocality functions. It offers an understanding of the relative speed of higher-frequency waves compared to lower-frequency waves. The influence of nonlocality, which is determined by the size of the horizon, can be comprehended by analyzing surface plots. As the value of δ approaches zero, it is observed that the dispersion relation of peridynamics converges to that of classical continuum mechanics. Anisotropic behaviour has significant implications for the design and analysis of structural components as it is widely available in nature, such as bone, wood, and rocks.

Keywords: Peridynamics, Anisotropic Material, Wave Dispersion, Green's Function, Nonlocal Elasticity.

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