Effects of suction and free stream velocity on hydromagnetic flow and heat transfer in a Newtonian fluid due to a stretching sheet and temperature dependent heat source(sink)

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1. INTRODUCTION AND OBJECTIVE

The study of suction and free stream velocity on hydromagnetic flow and heat transfer in a Newtonian fluid due to a stretching sheet and temperature dependent heat source has many industrial applications including aerodynamic extrusion of plastic sheets, condensation process of metallic plate in a cooling bath, fibre spinning, continuous molding, hot rolling, wire drawing, glass fibre and drawing of plastic films. The theoretical study of these applications by considering the boundary layer flow over a continuous solid surface moving with constant speed was initiated by Sakiadas [1–3].

Internal heat source(sink) can be used as an effective parameter to control convection. The influence of heat source(sink) in convection phenomenon is studied exhaustively (see Vanishree [4], Siddheshwar and Vanishree [5]). In the present work, we study the effect suction and free stream velocity in the presence of temperature dependent heat source (sink) on the boundary layer flow behavior and heat transfer of a Newtonian fluid past a linearly stretching or an exponentially stretching sheet in the presence of external magnetic field.

Keywords: Stretching sheet, Heat source(sink), Hydromagnetic flow, Heat transfer.

2. MATHEMATICAL FORMULATION

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\nu\frac{\partial^2 u}{\partial y^2}-\mu_m^2\sigma H^2 u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+Q'\left(T-T_0\right). \tag{4}$$

The following boundary conditions are employed on velocity

Linear stretching:
$$\left.\begin{aligned} u=u_0(x)=U_0\frac{x}{L}, v=v_w \text{ at } \ y=0,\\ u=u_\infty(x)=U_\infty\frac{x}{L}, \text{ as } \ y\to\infty. \end{aligned}\right\} \tag{5}$$

Exponential stretching: $u=u_w(x)=U_0 e^{S\frac{x}{L}}, v=v_w \ \text{ at } y=0.$ (6)

Equation (3) is solved subject to general isothermal boundary conditions

3. SOLUTION OF MOMENTUM EQUATION

3.1. Similarity solution of linear stretching sheet problem:

Using the similarity transformation and the choice of $A_l = B_l = 1,$ we get,

$$\left.\begin{aligned} g''' + gg'' - (g')^2 - Qg' + \left(Q\lambda_l + \lambda_l^2\right) = 0, \\ g(0) = -V_w,\ g'(0) = 1, g'(\infty) = \lambda_l. \end{aligned}\right\} \quad (7)$$

where, Q is the Chandrasekhar number and V_w is the suction/injection parameter.

3.2. Local similarity solution for the exponential stretching sheet problem:

Using the similarity transformation and the choice of $A_e = B_e = m = n = 1$ and $s = t = 2$, we get,

$$f''' + ff'' - 2(f')^2 - Q_x f' + \left(Q_x\lambda_e + 2\lambda_e^2\right) = 0, \quad (8)$$

$$\left.\begin{aligned} f(0) = -V_{wx},\ f'(0) = 1 \text{ at } \eta=0, \\ f'(\infty) = \lambda_e \text{ as } \eta \to \infty. \end{aligned}\right\} \quad (9)$$

where, Q_x is the local Chandrasekhar number and V_{wx} is the suction/injection parameter.

4. SOLUTION OF THE ENERGY EQUATION

The non-dimensional heat equation is solved subjected to non-isothermal boundary conditions-separate for linear and exponential stretching sheet

Linear stretching:

(i) Boundary with prescribed surface temperature (PST).

(ii) Boundary with prescribed heat flux (PHF).

Exponential stretching:

(i) Boundary with prescribed exponential order surface temperature (PEST).

(ii) Boundary with prescribed exponential order heat flux (PEHF).

4.1. *Pescribed surface temperature (PST)*

Introducing the dimensionless temperature, $\Theta_l(\eta) = \dfrac{\theta}{X^2}$ that satisfies

$$\Theta_l'' + \Pr g\,\Theta_l' + (R_I - 2g')\Pr\Theta_l + E_{PST}\Pr(g'')^2 = 0, \quad (10)$$

where $E_{PST} = \dfrac{\upsilon U_0 \operatorname{Re}}{C_p L(T_w - T_\infty)}$ is the Eckert number of the PST case.

4.2. *Prescribed heat flux (PHF)*

Introducing the dimensionless temperature, $\Phi_l(\eta) = \dfrac{\theta}{aX^2}$ we obtain

$$\Phi_l'' + \Pr g\,\Phi_l' + (R_I - 2g')\Pr\Phi_l + E_{PHF}\Pr(g'')^2 = 0, \quad (11)$$

where $E_{PHF} = \dfrac{\upsilon U_0 \operatorname{Re}}{C_p L(T_w - T_\infty)a}$ is the Eckert number of PHF case and $a = \dfrac{DL}{(T_w - T_\infty)\sqrt{\operatorname{Re}}}$.

4.3. *Prescribed exponential order surface temperature (PEST)*

In terms of $\Theta_e(\eta) = \dfrac{\theta}{e^X}$, we get

$$\Theta_e'' + \Pr f\,\Theta_e' + \left(R_I e^{2X} - f'\right)\Pr\Theta_e + E_{PEST}\Pr(f'')^2 = 0, \quad (12)$$

where $E_{PEST} = \dfrac{\upsilon U_0 \operatorname{Re} e^{3X}}{C_p L(T_w - T_\infty)}$ is the local Eckert number of PEST case.

4.4. *Prescribed exponential order heat flux (PEHF)*

Introducing the dimensionless temperature, $\Phi_e(\eta)=\dfrac{\theta}{be^{\frac{X}{2}}}$ we obtain

$$\Phi_e''+\Pr f\,\Phi_e'+\left(2R_I e^X-f'\right)\frac{\Pr}{2}\Phi_e+E_{PEHF}\Pr\left(f''\right)^2=0, \tag{13}$$

where $E_{PEHF}=\dfrac{\upsilon U_0 \operatorname{Re} e^{\frac{7X}{2}}}{C_p L\left(T_w-T_\infty\right)a}$ is the Eckert number of PEHF case and $b=\dfrac{T_1 L}{k\left(T_w-T_\infty\right)\sqrt{\operatorname{Re}}}$.

5. METHOD OF SOLUTION

The boundary layer flow generated by two types of stretching sheets are considered to study them. The nonlinear PDEs are converted to nonlinear ODEs by using the similarity transformation. Shooting method based on Runge Kutta45 and Newton-Raphson methods are used to obtain the equivalent IVPs corresponding to the BVPs. The Domb-Sykes plot is used to decide upon number of terms to be taken in the DTM. The accuracy chosen for the solution in the two problems is 10^{-6}.

6. CONCLUSIONS

i. On comparing the results on temperature of LSSP and ESSP, we may conclude that the results of ESSP are qualitatively similar to that of LSSP. LSSP facilitates faster rate of cooling compared to ESSP and hence is more suitable for imparting desired properties to the extrudate.

ii. PHF is better suited for faster cooling of the stretching sheet than the PST boundary condition. This is because boundary layer temperature is higher in PST case compared to PHF case.

iii. The effect of heat source (sink) parameter influences heating (cooling) of the stretching sheet.

iv. The dominant parameters, in terms of their influence on velocity and temperature distributions, are free stream parameter and suction/injection parameter. These two parameters have a pronounced effect in the cooling of the sheet.

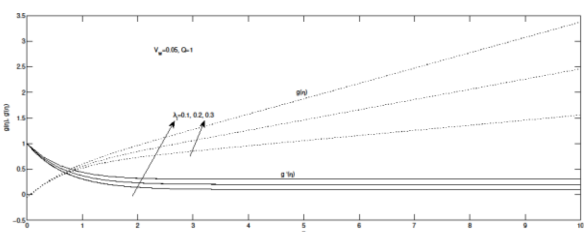

Fig. 1. Plots of $g(\eta)$ and $g'(\eta)$ versus η for the different values of suction/injection parameter V_w.

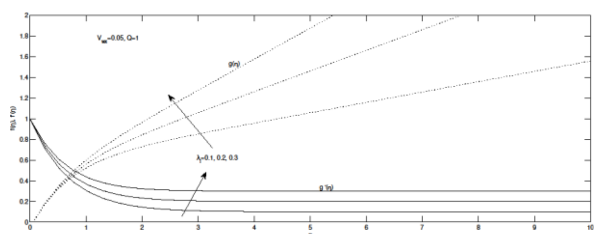

Fig. 2. Plots of $f(\eta)$ and $f'(\eta)$ versus η for the different values of suction/injection parameter V_w.

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