Spectral boundary integral equation method: Analysis of crack front waves

Yenike Sharath Chandra Mouli [1] and Ranjith Kunnath[2*],

[1]Post doctoral researcher, Mahindra University École Centrale School of Engineering, Hyderabad,

[2] Professor, Mahindra University École Centrale School of Engineering, Hyderabad.

1. INTRODUCTION

Crack front waves are persistent disturbances in the steadily propagating crack fronts that develop due to a local variation in material toughness. This work presents a numerical scheme for analysing crack front waves under mode-II slip rupture. A novel 3D spectral boundary integral equation method (BIEM) is adopted for the analysis. This method uses a boundary integral equation along the interfacial plane, taken in spectral form, to perform the space-time convolutions over the tractions, to obtain the displacement discontinuities. Unlike the numerical schemes available in the literature which takes the convolutions over the displacement discontinuities, we proposed a novel numerical scheme that uses the convolutions over the tractions. The numerical scheme originally developed for the simulation of 2D and 3D slip ruptures in the previous works. The current work is an additional validation to the numerical scheme through the analysis of crack front waves under the mode-II slip scenarios. The variation in the local toughness is achieved by varying the static and dynamic frictional coefficients at specific regions. A linearly degrading stick-slip cohesive law is applied for the computation of instantaneous shear fractional strength.

2. LITERATURE SURVEY

Crack front waves have been discovered recently during the numerical simulations carried out by Morrissey and Rice (1998), when a steadily propagating mode-I crack was made to pass over a local heterogeneity. A theoretical evidence for the crack front waves was shown by Ramanathan and Fisher (1997) following the framework of Willis and Movchan (1995). Fekak et al (2020) have simulated the 3D dynamic response of crack front waves under combined tensile and shear fracture modes. However, in the present work, the crack front wave behavior was analyzed for the mode-II slip rupture scenario. In this work, the 3D spectral BIEM originally developed by Gupta and Ranjith (2023) has been adopted for the analysis.

3. PROBLEM FORMULATION

Consider a planar interface between two identical isotropic homogeneous elastic half-spaces as shown in the Figure 1 with the fault plane having dimensions L_1=30,000m, L_3=15,000m. The asperity is positioned suitably such that the crack front reaches the asperity with a constant velocity. The material properties for elastic half-space of Gupta and Ranjith (2023) are used.

Further author information: (Yenike Sharath Chandra Mouli)
Y. Sharath Chandra Mouli: E-mail: sharathchandra.yenike@mahindrauniversity.edu.in, Ph. No: 9550410625.
Ranjith Kunnath: E-mail: ranjith.kunnath@mahindrauniversity.edu.in, Ph No. 9940649687.

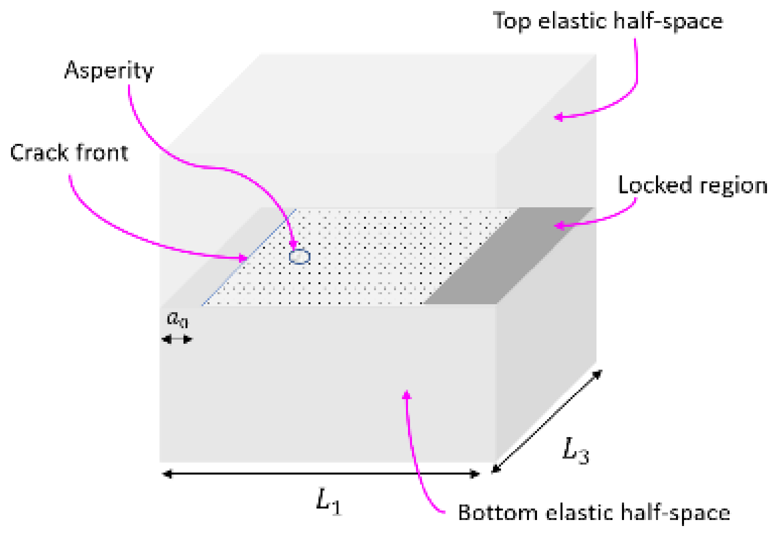

Figure 1. Model geometry with propagating crack front with constant speed v_0, before it reaches the asperity.

The crack front is simulated for 80% of the fault plane to avoid the reflections from the periodic boundary conditions. The fault plane is discretised into 1024x512 grid points and spectral BIEM is applied at the interface. As mentioned above, the stress components and the displacement discontinuities are represented in spectral form as follows:

$$\tau_j(x_1, x_3, t) = T_j(k, m, t)e^{ikx_1 + imx_3},\ j = 1,2,3 \tag{1}$$

$$\delta_j(x_1, x_3, t) = D_j(k, m, t)e^{ikx_1 + mx_3},\ j = 1,2,3 \tag{2}$$

The 3D spectral relations at the interface are given by following equations:

$$\begin{Bmatrix} \dfrac{\partial D_1(k,m,t)}{\partial t} + \left(\frac{c_s{}^+}{\mu^+} + \frac{c_s{}^-}{\mu^-}\right)T_1 \\ \dfrac{\partial D_2(k,m,t)}{\partial t} + \left(\frac{c_d{}^+}{\lambda^+ + 2\mu^+} + \frac{c_d{}^-}{\lambda^- + 2\mu^-}\right)T_2 \\ \dfrac{\partial D_3(k,m,t)}{\partial t} + \left(\frac{c_s{}^+}{\mu^+} + \frac{c_s{}^-}{\mu^-}\right)T_3 \end{Bmatrix} = \begin{Bmatrix} f_1(k,m,t) \\ f_2(k,m,t) \\ f_3(k,m,t) \end{Bmatrix} \tag{3}$$

where,

$$\begin{Bmatrix} f_1(k,m,t) \\ f_2(k,m,t) \\ f_3(k,m,t) \end{Bmatrix} = \int_0^t \begin{bmatrix} F_{11}(k,m,t') & F_{12}(k,m,t') & F_{13}(k,m,t') \\ F_{21}(k,m,t') & F_{22}(k,m,t') & F_{23}(k,m,t') \\ F_{31}(k,m,t') & F_{32}(k,m,t') & F_{33}(k,m,t') \end{bmatrix} \begin{Bmatrix} T_1(k,m,t-t') \\ T_2(k,m,t-t') \\ T_3(k,m,t-t') \end{Bmatrix} dt.$$

Here, the terms F_{ij}'s are the convolutions terms that are performed over the tractions. One can refer to Gupta and Ranjith (2023) for the complete expressions. An initial crack of length a_0

is applied with the nucleation stress for the slip to initiate. Later the loading was varied along the crack length as given in Fekak et al (2020).

4. RESULTS

Initially, the simulation is performed without the asperity in the fault plane to confirm the constant velocity in the propagation of crack front. The same loading is applied on the fault plane with the asperity of radius r=300m again. The slip variations at different equal time intervals compared with the case of no asperity. A detailed study of crack front waves has been carried out. This validates the versatility of the present numerical scheme.

5. CONCLUSIONS

A novel 3D spectral boundary integral equation method was applied on the two identical homogeneous elastic half-spaces to simulate the crack front waves. The results from the method are observed to study the crack front waves. This validates the versatility of the numerical scheme for the dynamic slip rupture problems.

References:

1. Fekak, F., Barras, F., Dubois, A., Spielmann, D., Bonamy, D., Geubelle, P. H., & Molinari, J. F. (2020). Crack front waves: A 3D dynamic response to a local perturbation of tensile and shear cracks. *Journal of the Mechanics and Physics of Solids*, *135*, 103806. https://doi.org/10.1016/j.jmps.2019.103806.
2. Gupta, A., & Ranjith, K. (2024). Spectral boundary integral equation method for simulation of 2D and 3D slip ruptures at bi-material interfaces. *International Journal for Numerical and Analytical Methods in Geomechanics*, *48*(1), 123–153. https://doi.org/10.1002/nag.3632.
3. Morrissey, J. W., & Rice, J. R. (1998). Crack front waves. *Journal of the Mechanics and Physics of Solids*, *46*(3), 467–487. https://doi.org/10.1016/S0022-5096(97)00072-0.
4. Ramanathan, S., & Fisher, D. S. (1997). Dynamics and instabilities of planar tensile cracks in heterogeneous media. *Physical Review Letters*, *79*(5), 877–880. https://doi.org/10.1103/PhysRevLett.79.877.
5. Willis, J. (1995). Dynamic weight functions for a moving crack. I. Mode I loading. *Journal of the Mechanics and Physics of Solids*, *43*(3), 319–341. https://doi.org/10.1016/0022-5096(94)00075-G.