Impact of heat source on thermal convection in viscoelastic ferromagnetic liquids with variable viscosity

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1. INTRODUCTION & OBJECTIVE

Convection is particularly important in industrial applications to study heat transport. The impact of variable viscosity on convection dynamics in viscoelastic ferromagnetic fluids offers a rich area of exploration with promising applications in renewable energy and environmental engineering. The study of thermal convection in Newtonian fluids, which are governed by Newton's law of viscosity, has been well-established for many years [1, 2]. In many industrial processes, liquid involved may be ferromagnetic, viscoelastic, or viscoelastic ferromagnetic. Ferromagnetic fluids combine the principles of magnetism and fluid dynamics, making them useful in a range of advanced technological and industrial applications [3], [4]. Viscoelastic fluids are characterized by their ability to exhibit both viscous and elastic behaviors, leading to unique flow properties and responses under different conditions [5].

It is well established that viscosity is influenced by both temperature and magnetic field. There are few works on thermal convection with viscosity dependent on temperature in viscoelastic fluid [6] & [7] and fewer works pertaining to convection in ferromagnetic fluids with temperature and magnetic field dependent viscosity [8] & [9].

Viscoelastic ferromagnetic fluids are non-Newtonian fluids having the properties of magnetic fluids and viscoelastic fluids. These materials exhibit unique behaviors due to their complex rheological properties, which can be tailored by external magnetic fields, temperature ([10], [11], [12], [13] & [14]). Few authors have studied convection in ferromagnetic/viscoelastic fluids with a heat source([15], [16] & [17]).

The literature highlights a gap in considering the effects of heat source on viscosity variation with temperature and magnetic fields. This study aims to investigate thermal convection in viscoelastic ferromagnetic fluids with variable viscosity and heat source using normal modes and Galerkin technique. Oscillatory convection is considered numerically under free-free isothermal and rigid-rigid isothermal boundaries

Governing Equations:

Basic equations

Continuity equation: $q_{i,i} = 0$

Momentum equation: $\rho_0 \frac{\partial q_i}{\partial t} = -\nabla p + \mu_0\left(\vec{M}.\nabla H\right) + \rho\overrightarrow{g_i} + \tau'_{ij,j}$

Constitutive equation: $(1 + \lambda_1 \frac{\partial}{\partial t})\, \tau'_{ij,j} = (1+\lambda_2 \frac{\partial}{\partial t})[\mu(H,T)[\frac{\partial \vec{q}_i}{\partial x_j} + \frac{\partial \vec{q}_j}{\partial x_i}]$

Energy Equation: $\frac{\partial T}{\partial t} + q_j T_{,j} = k[T_{,jj}] + Q_1(T - T_0)$

Where $q_i = (u, v, w)$ are the components of the velocity of the liquid, ρ is the density, $\rho_0\ is$ the density at the reference temperature T_0, p is the pressure, $\mu(T)$is the temperature dependent viscosity of the liquid, $g_i = (0,0,-g)$ are the components of the gravitational acceleration, $\lambda_1$is the stress relaxation coefficient, λ_2 is the strain retardation coefficient, T is the temperature and k is the thermal diffusivity. The density equation of state: $\rho = \rho_0(1 - \alpha(T - T_0))$ where $\alpha > 0$ is the constant of thermal expansion.

Maxwell's equations are $\nabla.B = 0$ and $\nabla X\vec{H}=0$, $\vec{B} = \mu_0[\vec{H}+\vec{M}]$

Magnetic equation of state is, $M = M_0 + \chi_m[H - H_0] - k_l[T - T_0]$

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Effective coefficient of viscosity $\mu(H,T)$ is given by $\mu(H,T)=\frac{\mu_0}{1+\delta_T(T-T_0)-\delta_H(H-H_0)}$

Perturbing basic states, nondimensionalizing with standard procedure and employing Galarkin technique to calculate the critical values of the Rayleigh number as:

R = X + iω Y

Since R is real, imaginary part equated to zero and an expression for ω^2 is obtained.

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

This work presents oscillatory convection in Jeffreys fluids with a stability analysis. The critical wave number a_c and frequency ω_c, which determine the critical Rayleigh number R_{oc} for the onset of convection, calculated across various parameter ranges, with the results summarized in Table 1. The table shows that for oscillatory convection, increasing Λ_1 and P_r leads to decrease in R_{oc} for both Maxwell and Jeffreys fluids. Conversely, an increase in $\boldsymbol{\Lambda}$ results in a higher R_{oc}. Additionally, we observe that R_{oc} decreases as V increases.

BC							V=0			V=0.3	
FIFI	M1	M3	Λ1	Λ2	Pr	Roc	ac	ωc	Roc	ac	ωc
	1	2	0.5	0.1	10	84.3774	2.86763	12.0974	74.9874	2.86783	11.69
	2	1	0.5	0.1	10	71.3311	3.09498	12.5904	62.5477	3.09471	12.1658
	1	1.1	0.5	0.1	10	92.5698	2.91623	12.2012	81.1708	2.91633	11.7905
Ri=-1	1	1	0.6	0.1	10	87.0055	2.87088	11.1837	76.3049	2.87026	10.8075
	1	1	0.5	0.2	10	157.799	2.80779	9.27863	138.914	2.80823	8.96015
	1	1	0.5	0.1	15	96.4125	2.92438	13.455	84.3145	2.92249	13.0848
	1	2	0.5	0.1	10	79.5462	2.86502	11.8904	69.3596	2.86349	11.4806
	2	1	0.5	0.1	10	67.1204	3.09293	12.3934	58.5306	3.09196	11.9687
	1	1.1	0.5	0.1	10	87.4083	2.9144	11.9979	76.2169	2.91293	11.5847
Ri=0	1	1	0.6	0.1	10	81.9197	2.8659	10.9956	71.4421	2.86347	10.617
	1	1	0.5	0.2	10	147.648	2.79569	8.95304	129.311	2.79443	8.73877
	1	1	0.5	0.1	15	91.1723	2.92356	13.1998	79.298	2.92038	12.8295
	1	2	0.5	0.1	10	74.901	2.8633	11.67	64.5564	2.86023	11.243
	2	1	0.5	0.1	10	63.0742	3.09168	12.1846	54.3733	3.09	11.7423
	1	1.1	0.5	0.1	10	82.438	2.91352	11.7817	71.0563	2.91062	11.3509
Ri=1	1	1	0.6	0.1	10	77.0247	2.86176	10.7954	66.3992	2.85746	10.3886
	1	1	0.5	0.2	10	137.855	2.78306	8.71604	119.427	2.78027	8.49521
	1	1	0.5	0.1	15	86.1112	2.92343	12.9291	74.0395	2.91911	12.5458

							V=0			V=0.3	
RIRI	M1	M3	Λ1	Λ2	Pr	Roc	ac	Ωc	Roc	ac	ωc
	1	2	0.5	0.1	10	182.681	3.9315	16.2004	159.878	3.92547	15.7708
	2	1	0.5	0.1	10	144.369	4.15681	16.6592	126.35	4.15036	16.2064
	1	1.1	0.5	0.1	10	196.406	4.01087	16.3606	171.91	4.00458	15.9226
Ri=-1	1	1	0.6	0.1	10	186.885	3.96026	14.9616	163.673	3.95403	14.5646
	1	1	0.5	0.2	10	340.739	3.84069	11.8288	299.795	3.8398	11.6332
	1	1	0.5	0.1	15	204.101	4.03704	17.6471	178.637	4.03308	17.282
	1	2	0.5	0.1	10	171.945	3.92374	15.9661	149.502	3.92168	15.5399
	2	1	0.5	0.1	10	135.504	4.15	16.44	117.832	4.14784	15.989
	1	1.1	0.5	0.1	10	185.117	4.00419	16.133	160.962	4.00205	15.6982

Ri=0	1	1	0.6	0.1	10	175.821	3.95078	14.7494	152.982	3.94849	14.3552
	1	1	0.5	0.2	10	319.037	3.82238	11.5949	279.014	3.82447	11.4063
	1	1	0.5	0.1	15	192.499	4.02992	17.3726	167.387	4.03049	17.0147
	1	2	0.5	0.1	10	161.6	3.91603	15.7209	138.78	3.91705	15.2811
	2	1	0.5	0.1	10	126.987	4.14319	16.2112	109.048	4.14434	15.7433
	1	1.1	0.5	0.1	10	174.226	3.99759	15.8958	149.62	3.99867	15.4461
Ri=1	1	1	0.6	0.1	10	165.157	3.94119	14.5272	141.94	3.9419	14.12
	1	1	0.5	0.2	10	298.133	3.8031	11.3498	257.722	3.80827	11.1603
	1	1	0.5	0.1	15	181.292	4.02272	17.0869	155.71	4.0269	16.7217

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