# Gravity wave interaction with a floating elastic plate in the presence of a pair of porous arc walls

P. Negi [1], T. Sahoo[1], and M. H. Meylan[2]

[1]Department of Ocean Engineering and Naval Architecture, Indian Institute of Technology Kharagpur, 721302, India
[2]School of Information and Physical Sciences, The University of Newcastle, Callaghan NSW 2308, Australia

## 1 INTRODUCTION & OBJECTIVE

In recent times, there have been significant breakthroughs in the study of surface gravity wave interaction with very large floating structures (VLFS) of various forms and geometries. Additionally, these structures are proposed and constructed for floating airports, floating offshore bases, storage facilities, and sustainable uses of ocean space to promote the blue economy. Among floating structures of various configurations, circular structures are preferred for optimum utilisation of ocean space. Furthermore, porous breakwaters are generally constructed to mitigate wave-induced forces on floating structures and create a calm zone in the vicinity of the floating facility. In recent years, arc-shaped porous structures have been proposed to improve protective effects and reduce the construction costs of complete porous cylindrical structures. Zhai et al. [3] analysed the diffraction problem of the interaction of solitary waves with the combined asymmetric porous arc walls sheltering an impermeable cylinder. Further, a recent study depicts that flexural gravity wave blocking may occur when the floating elastic plate is under the action of higher lateral compressive stress. In such a situation, the blocking/saddle point occurs, where the dispersion relation possesses roots of multiplicity two/three for certain wave frequencies. Das et al. [2] discovered that the dispersion relation of flexural gravity waves has three propagating wave modes within two different blocking points for certain fixed values of compressive force and frequency. In the present study, a mathematical model is developed to study the interaction of surface gravity waves with the dual porous arc-shaped bottom-mounted breakwater (ABBW) enclosing a circular floating elastic plate. The role of flexural gravity wave blocking in gravity wave interaction with a circular floating elastic plate is studied in the presence of a pair of porous arc walls. Fourier-Bessel series type expansion formulae are used to account for single as well as multiple propagating wave modes. Various hydrodynamic characteristics such as the hydrodynamic forces and moments acting on the arc walls and the elastic plate are analysed.

## 2 SOLUTION METHOD AND IMPORTANT RESULTS

In the present study, a mathematical model is developed to study the interaction of surface gravity waves with the dual porous arc-shaped bottom-mounted breakwater (ABBW) enclosing a circular floating elastic plate. The physical problem is considered in a three-dimensional cylindrical polar coordinate system $(r, \theta, y)$ with $r - \theta$ being the horizontal plane, as shown in Fig. 1. The outer and inner ABBW are symmetrically placed between $\theta = \alpha_{1,2}$ and $\theta = 2\pi - \alpha_{1,2}$ at a radial distances $r = a, b$. In further consideration, a circular elastic plate with radius $r = c$ is positioned to align its centre with the origin $O$. Therefore, the fluid domain is divided into four sub-domains: $\Omega_j$ for $j = 1, 2, 3, 4$. Moreover, the water depth is assumed to be finite $h$, and the incident wave reaches the breakwaters, creating an angle $\beta$ with the $x-$axis, as shown in Fig. 1. Furthermore, it is assumed that the elastic plate is acted on by the uniform lateral compressive force $\mathcal{N}$. Moreover, the fluid is assumed to be inviscid and incompressible, whereas the flow is considered to be irrotational

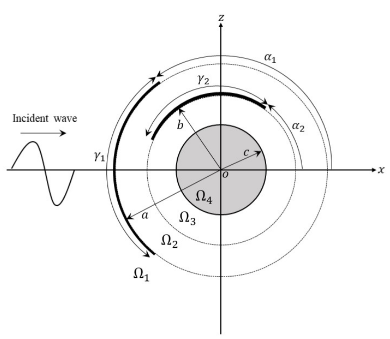



Figure 1: Schematic diagram of wave interaction with dual arc walls

and simple harmonic in time with angular frequency $\omega$. Thus, there exists a velocity potential and surface displacements of the forms $\Phi_j(r,\theta,z,t) = \Re\{\phi_j(r,\theta,z)e^{-\mathrm{i}\omega t}\}$ and $\zeta_j(r,\theta,t) = \Re\{\eta_j(r,\theta)e^{-\mathrm{i}\omega t}\}$ respectively. Thus, the spatial velocity potentials $\phi_j$s satisfy

$$\left(\nabla_{r\theta}^2 + \partial_y^2\right)\phi_j = 0, \quad \text{for} \quad j = 1,2,3,4, \tag{1}$$

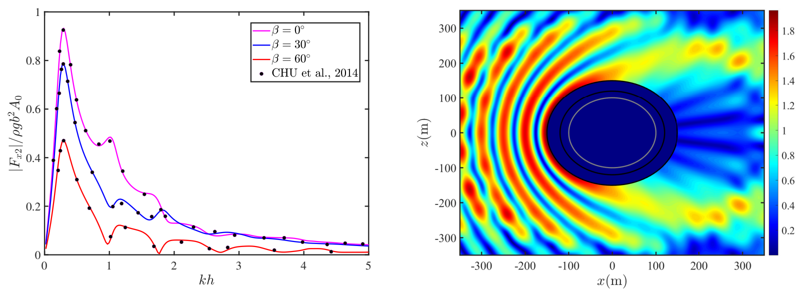
where $\nabla_{r\theta}^2 \equiv \partial_r^2 + (1/r)\,\partial_r + (1/r^2)\,\partial_\theta^2$. Further, we present the form of the velocity potentials using the Fourier-Bessel type expansion formula. The spatial velocity potentials $\phi_j(r,\theta,y)$ are given by

$$\begin{aligned}\phi_1(r,\theta,y) = &\frac{-\mathrm{i}gH}{2\omega}\sum_{m=0}^{\infty} f_0(y)\epsilon_m \cos m(\theta-\beta)J_m(k_0 r)\\ &+\sum_{i=0}^{\infty}\sum_{m=0}^{\infty} f_i(y)\left(A_{m,i}^{(1)}\cos m\theta + B_{m,i}^{(1)}\sin m\theta\right)H_m^{(1)}(k_i r),\end{aligned} \tag{2}$$

$$\begin{aligned}\phi_j(r,\theta,y) = &\frac{-\mathrm{i}gH}{2\omega}\sum_{i=0}^{\infty}\sum_{m=0}^{\infty} f_i(y)\left(A_{m,i}^{(j)}\cos m\theta + B_{m,i}^{(j)}\sin m\theta\right)J_m(k_i r)\\ &+\sum_{i=0}^{\infty}\sum_{m=0}^{\infty} f_i(y)\left(C_{m,i}^{(j)}\cos m\theta + D_{m,i}^{(j)}\sin m\theta\right)H_m^{(1)}(k_i r) \text{ for } j = 2,3,\end{aligned} \tag{3}$$

$$\phi_4(r,\theta,y) = \frac{-\mathrm{i}gH}{2\omega}\sum_{i=0,I,II,1}^{\infty}\sum_{m=0}^{\infty} g_i(y)\left(A_{m,i}^{(4)}\cos m\theta + B_{m,i}^{(4)}\sin m\theta\right)J_m(p_i r), \tag{4}$$

where $J_m(.)$ and $H_m^{(1)}(.)$ are the Bessel function and Hankel function of the first kind of order $m$ respectively. Further, $A_{m,i}^{(j)}$, $B_{m,i}^{(j)}$, $C_{m,i}^{(2)}$, $D_{m,i}^{(2)}$, $C_{m,i}^{(3)}$, and $D_{m,i}^{(3)}$ are the unknown complex constants which are to be determined. Besides, the velocity potential $\phi_4(r,\theta,y)$ will be generalized to account for multiple propagating wave modes during wave blocking and details will be discussed during the conference. For validation of the numerical results, hydrodynamics forces acting




(a) Force in $x-$direction (b) Flow pattern

Figure 2: (a) Validation and (b) verification of the physical model.

on the breakwater are computed and compared with the known results of a single rigid arc-shaped bottom-mounted breakwater ([1]). The horizontal force $|F_{x2}|/\rho g b^2 H$ as in Fig. 2a is plotted for different values of the incident wave angle $\theta$. A good agreement is observed between the present results and that of [1] which depicts that our analytical model will be useful and effective for studying the present problem related to different parameters. Subsequently, a pair of rigid cylinders is considered around the floating elastic disk under the porous condition $G_1 = G_2 = 0$ and the angles $\alpha_1 = \alpha_2 = 0°$. The flow distributions around the cylinders and the disk are exhibited in Fig. 2b. It is observed that the incident wave is diffracted around the outer cylinder and that there is no wave or plate excitation in the annular region between the two cylinders. Moreover, various results on the role of the arc-shaped breakwaters in mitigating the wave-induced forces on the floating plate will be presented at the conference.

# References


[1] Y. C. Chu, J. S. Cheng, J. Q. Wang, Z. G. Li, and K. B. Jiang. Hydrodynamic performance of the arc-shaped bottom-mounted breakwater. China Ocean Engineering, 28:749–760, 2014.

[2] S. Das, T. Sahoo, and M. H. Meylan. Dynamics of flexural gravity waves: from sea ice to hawking radiation and analogue gravity. Proc Roy Soc Ser A, 474(2209):20170223, 2018.

[3] Z. Zhai, S. Zheng, and D. Wan. Interaction between solitary waves and a combined structure of two concentric asymmetric porous arc walls. Physics of Fluids, 34(4), 2022.