Non-Linear Magnetoconvection in a Weakly Electrically Conducting Rotating Viscoelastic Liquid with Gravity Modulation

Suma N Manjunath[a] , Jayalatha G[b] , P G Siddheshwar[c] and T N Sakshath[d]
[a,b & d] RV College of Engineering, Bengaluru, India
[c] Centre of Mathematical Needs, Christ University, Bengaluru, India

INTRODUCTION & OBJECTIVE

Rayleigh-Benard convection (RBC) is a particular form of natural convection that arises when a fluid layer is heated from below and cooled from above, creating temperature gradients that generate convection currents. Rotation provides the liquid, an elastic behavior, and hence it is of great interest to include the impact of rotation on the instability of a liquid layer heated from beneath. In industrial applications, most commonly used non- Newtonian liquids are found to be viscoelastic liquids. Some important examples of viscoelastic liquids are paints, glues, and tars. Natural convection in viscoelastic liquids is the physical mechanism used in some of the applications, namely, fabrication of corrugated surfaces and deoxyribonucleic acid (DNA) replication.

Numerous authors have investigated transfer of heat and convection of Rayleigh-Benard type driven by surface tension as well as buoyancy in liquids of viscoelastic type namely Maxwellian, Oldroyd's has been addressed by various authors(see Sokolov and Tanner [1], Sekhar & Jayalatha [2, 3], Siddheshwar et al. [4] & Jayalatha & Suma [5]). These authors (see Bhatia & Steiner [6]) have taken a keen interest in studying the impact of the Coriolis force on the stability of a liquid layer, given that the rotation induces behavior reminiscent of elasticity.

Khayat [7] conducted a comprehensive study on the turbulent convection and unpredictable dynamics of the Oldroyd liquid B. Numerous researchers have explored convection phenomena under the influence of modulated temperature and gravity (see Venezian [8], Siddheshwar et al. [9] and Jayalatha & Suma [10]). Bhatia and Steiner [11], Takashima [12] have specifically studied magnetoconvection in viscoelstic liquids with non variable viscosity.

MRI machines operate in environments with significant mechanical vibrations and acoustic noise, which can degrade image quality and affect patient comfort has been studied by McJury and Shellock [13].

In view of the above situations, the present study focuses on the impact of gravity modulation on RBC and heat transfer in a rotating viscoelastic liquid of weakly electrically conducting type that are not previously been explored.

The objective of the paper is to investigate the following aspects in the current problem:

1. Studying the onset of convection and amount of heat transport in a weakly electrically conducting and rotating viscoelastic liquid in the presence of magnetic field with gravity modulation using free–free, isothermal boundaries.
2. Predicting the amount of heat transport in the system by varying various non-dimensional parameters.
3. Investigating the amount of heat transport in Oldroyd, Newtonian, Maxwell and Rivlin-Ericksen liquids by considering appropriate limiting cases.
4. Investigating the impact of combined effects of magnetic field and gravity modulation on amount of heat transport.

Governing Equations:
Basic equations
Continuity equation: $q_{i,i} = 0$
Momentum equation:
$$\rho_0 \frac{\partial q_i}{\partial t} + q_j q_{i,j} = -\nabla p + \frac{\rho_0 |\overrightarrow{\Omega_1} \times \vec{r}|^2}{2} + \rho g_i + \tau'_{ij,j} + 2\rho_0 \epsilon_{ijk} q_i \Omega_{1k} - \sigma_1 \mu_m^2 H_0^2 q_i$$
Constitutive equation: $(1 + \lambda_1 \frac{\partial}{\partial t})\, \tau'_{ij,j} = \mu\, (1+\lambda_2 \frac{\partial}{\partial t})[\frac{\partial \vec{q}_i}{\partial x_j} + \frac{\partial \vec{q}_j}{\partial x_i}]$
Energy Equation: $\frac{\partial T}{\partial t} + q_j T_{,j} = \kappa [T_{,jj}]$
Density Equation: $\rho = \rho_0 [1 - \alpha (T - T_0)]$

Where $q_i = (u, v, w)$ are the components of the velocity of the liquid, ρ is the density, ρ_0 is the density at the reference temperature T_0, p is the pressure, μ is the constant viscosity of the liquid, $g_i = (0,0,-g_0(1 + \epsilon \cos \omega t))$ are the components of the gravitational acceleration, $\lambda_1$is the stress relaxation coefficient, λ_2 is the strain retardation coefficient, T is the temperature and κ is the thermal diffusivity. The density equation of state: $\rho = \rho_0 (1 - \alpha (T - T_0))$ where $\alpha > 0$ is the constant of thermal expansion.

Perturbing basic states, nondimensionalizing with standard procedure and the highly coupled non-linear partial differential equations are reduced to Khayat-Lorenz model using stream functions, FIFI boundary conditions and appropriate trial functions. The resulting equations are solved by employing the built-in function Runge-Kutta-Fehlberg45 (RKF45) method available in Wolfram Mathematica 9. The effectiveness of heat transport is quantified using the Nusselt Number (*Nu*).

RESULTS AND HIGHLIGHTS OF IMPORTANT POINTS

This work studies the behavior of rotating viscoelastic liquid under the influence of magnetic fields and oscillatory forces. From the following table, it is clear that the heat transport decreases with increase in modulation amplitude in the absence of magnetic field and decreases in the presence of magnetic field.

	$Q = 0, Ta = 0$	$Q = 100, Ta = 0$	$Q = 0, Ta = 10$	$Q = 100, Ta = 10$
$\epsilon = 0$	2.31182	2.81677	2.33927	2.79445
$\epsilon = 0.1$	2.28062	2.88658	2.28648	2.87549

It has been observed that mean Nusselt number decreases as Λ_1, Λ increase in the absence magnetic field and as Ta increases in the presence of magnetic field there by advancing the convection. Also, with increase in Λ in the presence of magnetic field and Ta in the absence of the same, it is found that the heat transport is enhanced there by delaying the convection. The heat transport is much higher in the presence of magnetic field than in the absence of the same with increase in Prandtl number.

REFERENCES

1. Sokolov, M., Tanner, R.I.: Convective stability of a general viscoelastic fluid heated from below, Phy. Fluids.,15,534-539(1972)
2. Sekhar, G.N., Jayalatha, G.: Elastic effects on Rayleigh-B´enard-Marangoni convection in liquids with temperature-dependent viscosity, Proc. ASME 2009 IMECE09.,1830,978-07918-3863-1(2009)
3. Sekhar, G.N., Jayalatha, G.: Elastic effects on Rayleigh-B´enard convection in liquids with temperature-dependent viscosity, Int. J. of Therm. Sci.,49,67-75(2010)
4. Siddheshwar, P. G., Sekhar, G. N., Jayalatha, G.: Surface tension driven convection in viscoelastic liquids with thermorheological effect, Int. Commn. Heat and Mass transf.,38,468-473(2011)
5. Jayalatha, G., Suma, N.: Effect of Heat Source on Rayleigh-B´enard Convection in Rotating Viscoelastic Liquids, Heat Transfer,50,7672-7690(2021)
6. Bhatia, P.K., Steiner, J.M.: Convective instability in a rotating viscoelastic fluid layer, ZAMM.,52,321-327(1972)
7. Khayat, R.E.: Chaos and overstability in the thermal convection of viscoelastic fluids, J. Non-Newtonian Fluid Mech.,53,227-255(1994)
8. Venezian, G.I.: Effect of modulation on the onset of thermal convection, J. Fluid Mech.,35,243-254(1969)
9. Siddheshwar, P.G., Sekhar, G.N., Jayalatha, G.: Effect of time-periodic vertical oscillations of the Rayleigh B´enard system on nonlinear convection in viscoelastic liquids, J. Non- Newtonian Fluid Mech.,165,1412-1418(2010)
10. Jayalatha, G., Suma, N.: Nonlinear Rotating Viscoelastic Liquid Convection with Temperature Modulation, Heat Transfer, http://doi.org/10.1002/htj.21841(2020)
11. Bhatia, P.K., Steiner, J.M.: Convection in a viscoelastic fluid layer in hydromagnetics, Phys. Letters A,37,419-420(1971).
12. Takashima, M.: Thermal instability in a viscoelastic fluid layer. III. effect of a magnetic field, J. Phys. Soc. Japan,33,1142-1148(1972a).
13. McJury, M., and Shellock, F. G., Acoustic noise and MR procedures: A review. Journal of Magnetic Resonance Imaging, 12(1), 37-45 (2000).