Weakly non-linear stability analysis on gravity-modulated Rayleigh-Bénard convection in a Newtonian liquid bounded by rigid-free boundaries.

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1. ABSTRACT

The effect of gravity modulation on the post convective dynamical system was studied by considering a Newtonian fluid layer bounded by rigid-free isothermal boundaries. Weakly non-linear analysis was performed by considering second harmonics in the Fourier series expansion of temperature, while the second harmonics in the velocity field was neglected owing to the assumption of small-scale convective motions. Bifurcation diagram and Lyapunov exponents were calculated for varying combinations of modulation parameters. It was found that gravity-modulation has, in general, gravity modulation stabilizes the dynamical system by enhancing the efficiency of the natural frequency of the periodic convective motion. Another important outcome of the study is that hyperchaos was observed within the 3-dimensional system, emerging for varying, random combinations of frequency and amplitude values. This result is quite new.

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

The effect of gravity modulation on the post convective dynamical system was studied by considering a Newtonian fluid layer bounded by rigid-free isothermal boundaries. Weakly non-linear analysis was performed in which Bifurcation diagram and Lyapunov exponents were calculated for varying combinations of modulation parameters. The following observations were made:

1. The Hopf-Rayleigh number (rH) corresponding to the onset of chaos for rigid-free boundaries lies between free-free and rigid-rigid counterparts. This implies that gravity modulation can be adapted in practical situations to control the onset of chaos.
2. The effect of frequency of modulation is quite varied and depends on the magnitude of frequency on the fluid. Generally, however, frequency of modulation stabilizes the medium.
3. The effect of amplitude of modulation is to destabilize the system, in the transitions turbulence.
4. Gravity modulation induces hyperchaos within a tri-modal non-autonomous sys- tem, which is not associated with any specific regions of parameters spaces ($r\varepsilon$, $r\omega$ spaces).
5. Gravity modulation contributes to a vigorous-chaotic system.

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