Statistical framework for heat and mass transfer analysis of hybrid nanofluids in an annulus: A modified Buongiorno model

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1. ABSTRACT

In this modern fluid field technology, hybrid nanoliquid are of great interest to researchers because of their thermal properties which provide superior heat transfer improvements compared to nanoliquid. Thus, in this study, the heat and mass transport characteristics in a horizontal annular duct filled with the water-based Cu-Al_2O_3 hybrid nanoliquid is analyzed using the modified Buongiorno model (two-phase model). The two different heat sources namely, temperature-related heat source (THS) and exponential space-related heat source (ESHS) are analyzed in thermal analysis. An inclined magnetism and viscous dissipation aspects are also taken into account. The correlation for effective thermal conductivity and viscosity are modeled by utilizing the experimental work of Corcione. The coupled nonlinear equations were solved numerically using the finite difference method. Further, the heat transport rate is optimized using the response surface methodology (RSM). The significance of effective parameters on the flow structure, thermal pattern, concentration field, heat and mass transport rate are visualized through two-dimensional (2D) and three-dimensional (3D) surface plots.

2. RESEARCH AIM

This study highlights the heat and mass transport characteristics of hybrid nanoliquid (Cu-Al_2O_3- H_2O) in a horizontal annulus using modified Buongiorno model. The objectives of the present research are to explore the momentum and thermal behaviour of hybrid nanoliquid (Cu-Al_2O_3- H_2O) in the presence of two different heat sources (ESHS and THS), viscous dissipation and inclined magnetic field.

3. LITERATURE SURVEY

In order to improve heat transport, various types of nanoparticles are suspended into traditional liquids. The subsequent solution is referred to as "nanofluid" and the presence of nanoparticles in the liquid leads to an improvement in "thermal conductivity" and "heat transfer" (Choi and Eastman [1]). Recently, a new class of nanofluids that has caught the interest of researchers is the hybrid nanofluid, which comprises different types (two or more) of nanoparticles, such as Cu, Fe_2O_3, Al_2O_3, Fe_3O_4, CuO, TiO_2, SiO_2, Ag, carbon nanotubes (CNTs) and SiC. In contrast to mono nanofluids, hybrid nanofluids exhibit remarkable thermophysical-chemical properties and rheological performance. More importantly, hybrid nanofluids (HNF) can lead to reasonable and acceptable nanofluid stability. These stable nanoliquids have desirable properties, such as augmented thermal conductivity at small concentrations of nanoparticles. The hybrid nanofluids find their relevance in various applications such as electronic cooling, biomedical production, power systems, welding, vehicle thermal coordination, lubrication, and spacecraft. Experimental work to illustrate the significance of Al_2O_3-Cu hybrid nanoliquid on heat transfer rate was conducted by Suresh et al. [2]. It was proved that at the same volume concentration the thermal conductivity of HNF is higher than that of the mono nanoliquid. Recent works on hybrid nanofluid flow are elaborated in the references (see [3]-[4] and therein). However, the studies on hybrid nanoliquid flow using the modified Buongiorno model (two-phase approach) were extremely limited.

The effect of the magnetic field on a fluid flow has been widely studied due to its enormous advantages in the field of chemistry, engineering, and physics. With these factors in mind, the heat transfer in the hybrid nanoliquid exposed to the tilted magnetic field was inspected by Hayat et al. [5]. It was noticed that the friction factor decreases with an inclined angle of the magnetic field. The peristaltic flow of nanoliquid in an inclined concentric cylinder exposed to an inclined magnetic field was examined by Shahzadi and Nadeem [6]. However, the studies related to the influence of the inclined magnetic field on the flow of hybrid nanoliquid in a horizontal annulus using the modified Buongiorno model are very limited.

The Response Surface Methodology (RSM) is an integration of mathematical and statistical techniques utilized to construct experimental models. The main objective of this procedure is to optimize the dependent variable (response) that is affected by several independent variables. The magneto nanoliquid flow within a square enclosure through RSM was studied by Pordanjani et al. [7] and concluded that the rate of heat transfer is reasonably affected by magnetic force. Thriveni and Mahanthesh [34] investigated the combined effect of quadratic convection and nonlinear radiation in an annular duct filled with HNF using sensitivity analysis.

4. PROBLEM FORMULATION

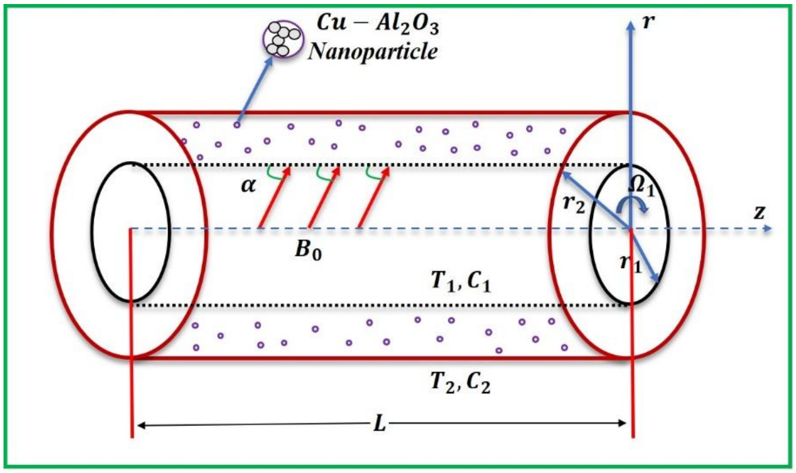

Fig. 1: Physical model.

The physical model and the assumed coordinate system are considered as illustrated in Fig. 1. The horizontal annular conduit of characteristics length L is filled with the water-based Cu-Al_2O_3 hybrid nanoliquid. The motion of hybrid nanoliquid is assumed to be non-transient, laminar, and incompressible. The modified Buongiorno model for the hybrid nanoliquid is considered along with the temperature-dependent heat source (THS), exponential space-related heat source (ESHS), and viscous dissipation phenomenon. A magnetic field of strength B_0 is applied at inclination angle α^0 with *z*-axis. The concentric cylinders of an outer radius r_2 and inner radius r_1, here the inner cylinder is rotated at a constant speed of Ω_1 by fixing the outer cylinder. The surface of the inner cylinder is maintained at an unequal constant temperature T_1

and concentration C_1 while T_2 and C_2 at the outer cylinders (where $T_1>T_2$ and $C_1>C_2$). Enforcing the above hypothesizes, the governing equations in the dimensional form are given below:

Conservation of linear momentum

$$\rho_{hnl}\, v\frac{\partial v}{\partial r} = \mu_{hnl}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)-\sigma_{hnl}B_0^2 v\, sin^2(\alpha), \tag{1}$$

Conservation of energy

$$\left.\begin{aligned}\left(\rho C_p\right)_{hnl} v\frac{\partial T}{\partial r} &= k_{hnl}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\mu_{hnl}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)^2+(T-T_2)Q_t\\ &\quad +(T_1-T_2)Q_e\exp\left(-\frac{nr}{r_2}\right)+\left(\rho C_p\right)_{hp}\left[D_B\frac{\partial C}{\partial r}\frac{\partial T}{\partial r}+\frac{D_T}{T_1}\left(\frac{\partial T}{\partial r}\right)^2\right],\end{aligned}\right\} \tag{2}$$

Concentration equation

$$v\frac{\partial C}{\partial r}=\frac{D_B}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)+\frac{D_T}{T_1}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{3}$$

Boundary conditions

$$\left.\begin{aligned}&\text{at}\quad r=r_1:\ \ v(r)=\Omega_1 r_1,\ \ C=C_1,\qquad T=T_1,\\ &\text{at}\quad r=r_2:\ \ v(r)=0,\qquad C=C_2,\qquad T=T_2.\end{aligned}\right\} \tag{4}$$

Introducing the following dimensionless quantities along with the thermophysical properties of hybrid nanoliquid:

$$\left.\begin{aligned}&R=\frac{r}{r_2},\ \ V=\frac{v}{\Omega_1 r_1},\ \ \eta=\frac{r_1}{r_2},\ \ \Delta C=C_1-C_2,\\ &\theta=\frac{T-T_2}{T_1-T_2}\ ,\ \ S=\frac{C-C_2}{C_1-C_2}\ ,\ \Delta T=T_1-T_2\end{aligned}\right\} \tag{9}$$

Eqns. (1)-(4) takes the form as follows:

$$\frac{\partial^2 V}{\partial R^2}+\frac{1}{R}\frac{\partial V}{\partial R}-\left[\frac{1}{R^2}+\frac{A_3}{A_2}\frac{Ha^2}{(1-\eta)^2}\sin^2\alpha\right]V-\frac{A_1}{A_2}ReV\frac{\partial V}{\partial R}=0, \tag{10}$$

$$\left.\begin{aligned}&\frac{1}{Pr}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)+\frac{A_2}{A_5}E_C\left(\frac{\partial V}{\partial R}-\frac{V}{R}\right)^2+\frac{Q_T}{A_5}\theta-\frac{A_4}{A_5}ReV\frac{\partial\theta}{\partial R}+\frac{Q_E}{A_5}\exp(-nR)\\ &\qquad +\frac{1}{Pr}\frac{Nb}{A_5}\frac{\partial S}{\partial R}\frac{\partial\theta}{\partial R}+\frac{1}{Pr}\frac{Nt}{A_5}\left(\frac{\partial\theta}{\partial R}\right)^2=0,\end{aligned}\right\} \tag{11}$$

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial S}{\partial R}\right)+\frac{Nt}{Nb}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)-ReScV\frac{\partial S}{\partial R}=0, \tag{12}$$

$$\left.\begin{aligned}&at\quad R=\eta:\ V(\eta)=1,\ S(\eta)=1,\ \theta(\eta)=1,\\ &at\quad R=1:\ V(1)=0,\ S(1)=0,\ \theta(1)=0.\end{aligned}\right\} \tag{13}$$

The physical quantities Nusselt number and Sherwood number at $R=\eta$ and $R=1$ are as follows:

At inner cylinder:

$$\left.\begin{aligned}&Nu_i=-\frac{k_{hnl}}{k_l}(1-\eta)\theta'(\eta),\\ &Sh_i=-(1-\eta)S'(\eta),\end{aligned}\right\} \tag{14}$$

At outer cylinder:

$$\left.\begin{aligned} Nu_o &= -\frac{k_{hnl}}{k_l}(1-\eta)\theta'(1), \\ Sh_o &= -(1-\eta)S'(1). \end{aligned}\right\} \quad (15)$$

Before employing the finite difference method based on the bvp4c algorithm, the system of nonlinear differential Eqns. (10)-(13) are converted into a system of first-order differential equations (ODEs) by substituting $[V, V', \theta, \theta', S, S'] = [y_1, y_2, y_3, y_4, y_5, y_6]$.

5. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

The significance of effective parameters on the flow structure, thermal pattern, concentration field, heat and mass transport rate are visualized through two-dimensional (2D) and three-dimensional (3D) surface plots. It is noticed that the chaotic motion of nanoparticles advances the thickness of the thermal and solutal boundaries. The velocity field has an inverse association with the applied magnetic field and its angle of inclination. The consequence of the Reynolds number is favorable for the velocity and temperature fields. The heat transport is more dominated by the Reynolds number compared to the chaotic motion of nanoparticles and thermophoretic aspect. Furthermore, the sensitivity of the Nusselt number to the Reynolds number, chaotic motion of nanoparticles and thermophoretic aspect are always negative.

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