Study of Chaos in a Two Component Rayleigh-Bénard Convection in a Micropolar Fluid

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1. INTRODUCTION & OBJECTIVE

Rayleigh-Bénard Convection (RBC) refers to the natural convective phenomenon in a Newtonian-Boussinesq fluid, induced by the temperature gradient between two horizontal parallel plates, with the lower plate heated and the upper plate cooled, leading to the development of thermal convection cells. Chandrasekar[1] explored this ancient problem in depth in his work. His work inspired several researchers to pursue investigations in this field.

In numerous industrial applications, fluids containing suspended particles serve as the working medium. These fluids fail to adhere to the Newtonian model, as the suspended particles demonstrate both translational and rotational motion relative to the fluid. Eringen[2] introduced the "micropolar-fluid-model" (MPF) to analyze flow and heat transmission in fluids containing suspended particles, using two uncommon variables: spin-variables that account for the micro-rotation vector and the micro-inertia tensor. This introduction results in a non-symmetric stress tensor and couple-stress in the classical Navier-Stokes equation. These fluids illustrate the behavior of manmade fluids, including colloidal suspensions, polymers, and fluids containing additives. Lukaszwicz[3], in his book on MPFs, has presented a rigorous mathematical framework for these fluids. Two applications of MPFs are described in the book: one in the theory of porous media and the other in the theory of lubrication. The proposed challenge is a prototype for circumstances happening in energy storage systems, thermal coolant systems and others. MPF are especially useful if one needs to increase the resistance time of heat in a system.

Yuan and Li[4] have studied the global regularity problem for the two-dimensional micropolar RBC system with zero velocity, micro-rotation and temperature dissipation using the Littlewood-Paley decomposition technique. The behaviour of the sub-critical area for MPF parameters in the presence of throughflow in a horizontal porous layer heated from below was examined by Barman and Srinivasacharya[5]. This study demonstrates that the sub-critical zone reduces when the parameter value drops, and there is no sub-critical gap when there is no throughflow. Pranesh and Arun[6] examined the influence of non-uniform concentration gradient on the initiation of DDC using the Galerkin method. They found that by choosing non-uniform concentration gradient onset of DDC can be regulated. No research have been done on the chaotic regime of two component RBC in MPFs, except Kanchana et al.[7] ,who explored chaotic motion in a single component RBC in a Newtonian fluid.

Governing Equation

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p - \rho g \hat{k} + (2\zeta + \eta)\nabla^2 \vec{q} + \zeta \nabla \times \vec{\omega} \tag{2}$$

$$\rho_0 I \left[\frac{\partial \vec{\omega}}{\partial t} + (\vec{q}.\nabla)\vec{\omega}\right] = (\lambda' + \eta')\nabla(\nabla.\vec{\omega}) + \eta' \nabla^2 \vec{\omega} + \zeta(\nabla \times \vec{q} - 2\vec{\omega}) \tag{3}$$

$$\frac{\partial T}{\partial t} + \left[\vec{q} - \frac{\beta}{\rho_0 C_v} \nabla \times \vec{\omega}\right].\nabla T = \chi \nabla^2 T \tag{4}$$

$$\frac{\partial C}{\partial t} + \vec{q}.\nabla C = \chi_C \nabla^2 C \tag{5}$$

$$\rho = \rho_0 [1 - \alpha_t (T - T_0) + \alpha_C (C - C_0)] \tag{6}$$

Stationary Rayleigh number

$$\boldsymbol{R_S} = \left(\frac{\boldsymbol{K^6}}{\boldsymbol{\pi^2 \alpha^2}}\right) \boldsymbol{F} + \boldsymbol{Q} \frac{\boldsymbol{R_C}}{\boldsymbol{\tau}} \tag{7}$$

Where, $\boldsymbol{F} = \frac{\boldsymbol{(1+N_1) N_3 K^2 + 2N_1 + N_1^2}}{\boldsymbol{N_3 K^2 + 2N_1}}$, $Q = \frac{N_3 K^2 + 2N_1 - N_1 N_5 K^2}{N_3 K^2 + 2N_1}$

Scaled Lorenz model

$$\frac{dX_1}{d\tau_1} = \widetilde{Pr}[X_3 - X_1 - N_1 X_2 + X_5] \tag{8}$$

$$\frac{dX_2}{d\tau_1} = -\frac{\widetilde{Pr}}{M_1}[N_1 X_1 + M_2 X_2] \tag{9}$$

$$\frac{dX_3}{d\tau_1} = \tilde{r} X_1 - X_3 - X_1 X_4 + N_5 M_3 (\tilde{r} X_2 - X_2 X_4) \tag{10}$$

$$\frac{dX_4}{d\tau_1} = -b X_4 + X_1 X_3 + N_5 M_3 X_2 X_3 \tag{11}$$

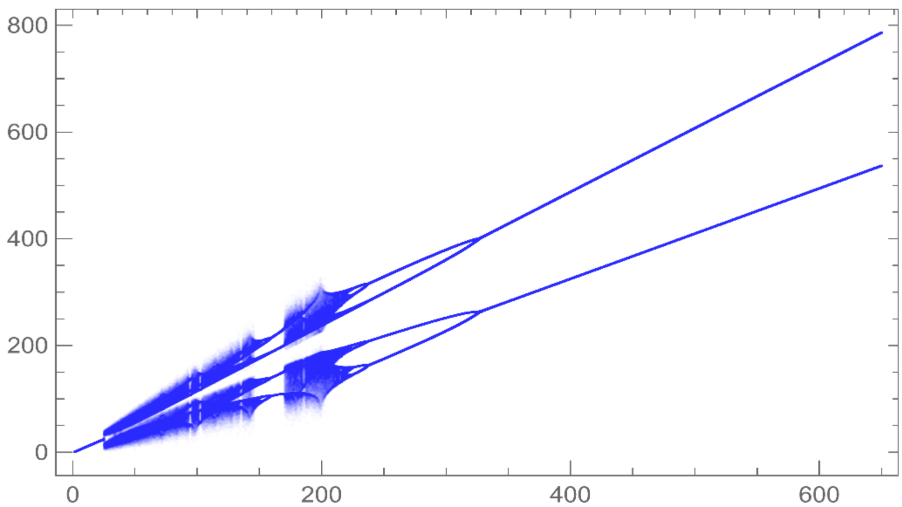
$$\frac{dX_5}{d\tau_1} = -\tau X_5 - 2 X_1 X_6 - M_4 R_C X_1 \tag{12}$$

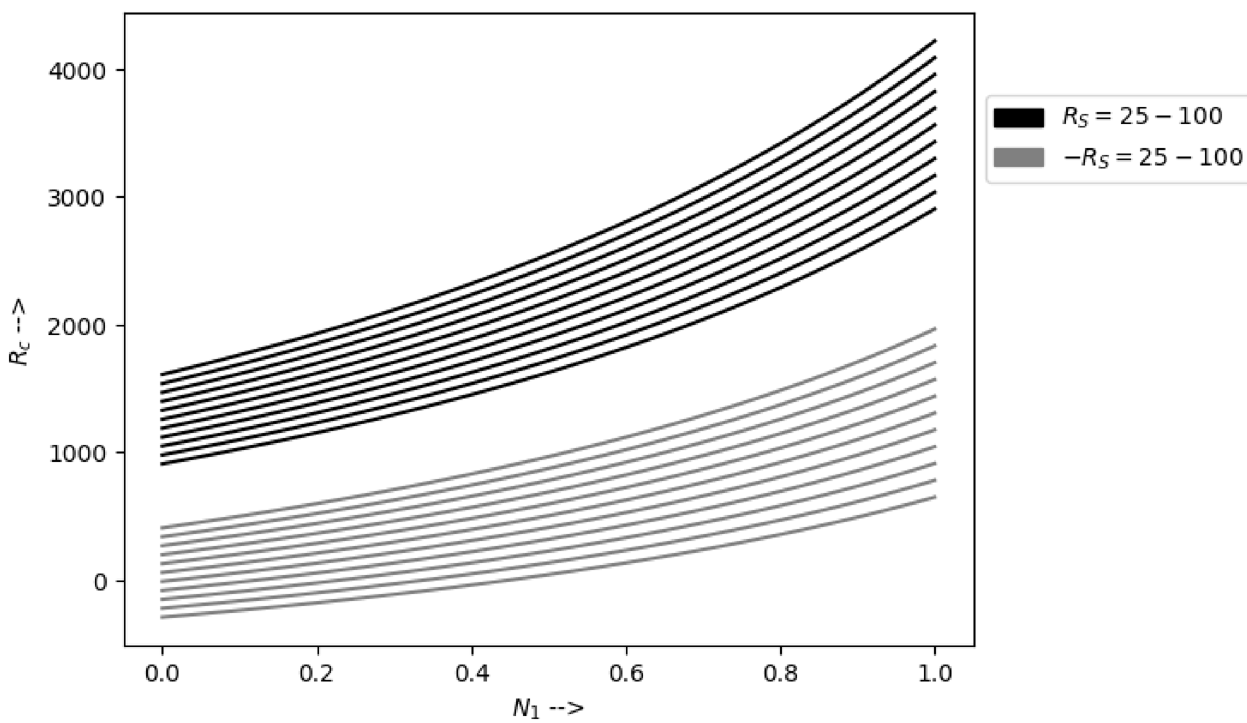
$$\frac{dX_6}{d\tau_1} = -b\tau X_6 + X_1 X_5 \tag{13}$$

Where, $M_1 = K^2 N_2 (1 + N_1)^2, M_2 = (1 + N_1)(K^2 N_3 + 2N_1), M_3 = K^2 (1 + N_1)$,

$M_4 = \frac{\pi^2 \alpha^2}{K^6 (1 + N_1)}$.

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS





1. Increase in the parameter values of N_1 and N_2 delays the onset of regular convection and onset of chaotic motion.
2. Increase in parameter N_3 advances the onset of regular convection and onset of chaotic motion.
3. Parameter N_5 has negligible effect on the onset of chaos, hence it is neglected for further studies.
4. Increase in parameter R_S delays the onset of regular convection and onset of chaotic motion.

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