Linear Stability Analysis of Triple Diffusive Convection in the presence of Internal Heat in Newtonian Fluids.

Kavya S Kumar[a], S Pranesh[b] and Arun Surya E S[c]

[a]Centre for Mathematical Needs, CHRIST(Deemed to be University), Banglore, India
[b]Centre for Mathematical Needs, Faculty, CHRIST(Deemed to be University), Banglore, India
[c]Centre for Mathematical Needs, CHRIST(Deemed to be University), Banglore, India

Abstract

In this paper, investigates the dynamics of Rayleigh–Bénard convection with two solutes in Newtonian fluids in the presence of an internal heat source are investigated. A linear stability analysis is performed using the Galerkin method, leading to the expression for the Rayleigh number, which indicates the onset of convection. Sixteen boundary conditions are explored to analyse the stability of the system under different non-dimensional parameters. The graphical representations provide insights into the system's behaviour under the influence of these parameters. The findings contribute to a deeper understanding of fluid convection and its linear stability characteristics.

1. INTRODUCTION

Rayleigh–Bénard convection is a natural convection which facilitates the transfer of heat. Density difference and gravitation force affect the convection. Triple-component convection is the study of multi-component convection which involves three components with different individualistic properties. There will be two salts s_1 and s_2 present in the system with concentrations C_1 and C_2. Conservation of soluted concentration is also considered among the other governing equations. The effect of the multi-component convection has been studied in several fields such as petroleum industries, chemical engineering and Geothermal regions. Natural convection is driven by internal heating like in atmospheres and mantle earth and it can be observed that the heat generated is not uniform. Internal heating can also be induced in man-made objects like induction heaters and microwave ovens and it can be similarly observed that the heat generated is not uniform. Though internal heating plays a vital role in convection, it is mostly neglected.

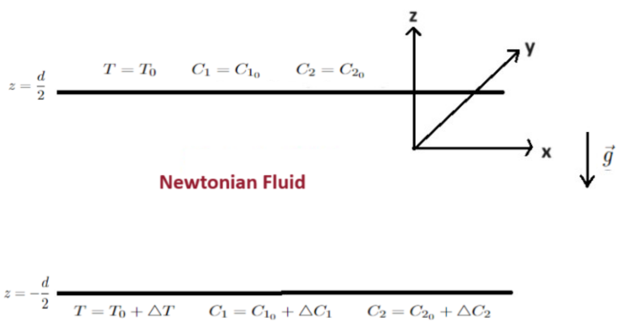

Figure 1: **Physical configuration.**

Consider an infinitely extended horizontal layer of Newtonian fluid of uniform vertical thickness d. Let the lower plate and the upper plate be kept at $z = -\frac{d}{2}$ and $z = \frac{d}{2}$ respectively. Let the boundary layer be heated from below (to the lower plate) with temperature given by $T = \triangle T + T_0$ and the difference between temperatures of both the plates are $\triangle T$, i.e., the temperature at the upper plate is $T = T_0$. Let $\triangle C_1$ and $\triangle C_2$ be the concentration difference between the layers for solutes s_1 and s_2 respectively. The lower and upper boundaries be placed at $z = \frac{-d}{2}$ and $z = \frac{d}{2}$ respectively. Gravity is assumed to be acting vertically downwards. The physical configuration is shown in Figure 1.

2. GOVERNING EQUATIONS

With Boussinessquian approximation, viscosity and thermal conductivity took constant, density-dependent on temperature, the following governing equations are used to modulate the problem.

1. Equation of continuity

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

2. Conservation of Linear Momentum

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} \right] = -\nabla p - \rho g \hat{k} + \mu \nabla^2 \vec{q}, \tag{2}$$

3. Conservation of Energy (with internal heat)

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \chi \nabla^2 T + Q(T - T_0), \tag{3}$$

4. Conservation of soluted concentration 1

$$\frac{\partial C_1}{\partial t} + (\vec{q} \cdot \nabla)C_1 = \chi_{s_1} \nabla^2 C_1, \tag{4}$$

5. Conservation of soluted concentration 2

$$\frac{\partial C_2}{\partial t} + (\vec{q} \cdot \nabla)C_2 = \chi_{s_2} \nabla^2 C_2, \tag{5}$$

6. Equation of state

$$\rho = \rho_0 \left[1 - \alpha_t (T - T_0) + \alpha_{s_1}(C_1 - C_{1_0}) + \alpha_{s_2}(C_2 - C_{2_0}) \right], \tag{6}$$

where, $\vec{q}$ is the velocity of fluid, T is the temperature, p is the pressure, μ is viscosity, χ is the thermal conductivity, Q is the heat source, χ_{s_1} and χ_{s_2} are the solute diffusivities respectively.

3. METHODOLOGY

Consider the basic state for this problem where the basic velocity is 0 and other variables to be a function of the vertical component. Also, the perturbed states are considered as well. We get coupled equations using the governing equations 1–6 in both basic and perturbed states. To simplify, the number of parameters is reduced by non-dimensionalisation. The dimensionless parameters are Rayleigh number R, internal Rayleigh number Ri, Solute Rayleigh numbers R_{s_1} and R_{s_2} and ratio of diffusivities τ_1 and τ_2. The expression of Rayleigh number R is

$$R = \frac{\rho_0 \alpha_t \triangle T d^3 g}{\mu \chi}. \tag{7}$$

When the amplitude of perturbation is minimal, to the extent that we can consider them to be infinitesimal, then the products and the powers of such perturbations are negligibly small compared to the linear terms. In such a case, we can discard the nonlinear terms and thus 'linearize' the system of equations. Further using the vortex method, we get coupled partial differential equations.

Using normal mode analysis and solving the coupled equation, the expression for R is obtained which determines the onset of convection. Here, we consider the growth rate to be zero in normal mode analysis and hence we get stationary convection. The value of the critical Rayleigh number depends on the boundary conditions for the problem. For temperature T, there are two cases isothermal and adiabatic. Let us consider isoconcentration for both the solutes s_1 and s_2. Therefore,

$$\phi_{11}(z) = 4z^2 - 1 \tag{8}$$

$$\phi_{21}(z) = 4z^2 - 1 \tag{9}$$

The 16 boundary conditions used for analysis are listed in the Table 1.

Table 1: **List of trial functions for 16 boundary combinations.**

Sl No.	Boundary Combination	W_1 (velocity)	T_1 (temperature)
1	Free Isothermal Free Isothermal	$16z^4 - 24z^2 + 5$	$4z^2 - 1$
2	Free Adiabatic Free Isothermal	$16z^4 - 24z^2 + 5$	$4z^2 + 4z - 3$
3	Free Isothermal Free Adiabatic	$16z^4 - 24z^2 + 5$	$4z^2 - 4z - 3$
4	Free Adiabatic Free Adiabatic	$16z^4 - 24z^2 + 5$	1
5	Free Isothermal Rigid Isothermal	$8z^4 + 4z^3 - 6z^2 - z + 1$	$4z^2 - 1$
6	Free Adiabatic Rigid Isothermal	$8z^4 + 4z^3 - 6z^2 - z + 1$	$4z^2 + 4z - 3$
7	Free Isothermal Rigid Isothermal	$8z^4 + 4z^3 - 6z^2 - z + 1$	$4z^2 - 4z - 3$
8	Free Adiabatic Rigid Adiabatic	$8z^4 + 4z^3 - 6z^2 - z + 1$	1
9	Rigid Isothermal Free Isothermal	$8z^4 - 4z^3 - 6z^2 - z + 1$	$4z^2 - 1$
10	Rigid Adiabatic Free Isothermal	$8z^4 - 4z^3 - 6z^2 - z + 1$	$4z^2 + 4z - 3$
11	Rigid Isothermal Free Adiabatic	$8z^4 - 4z^3 - 6z^2 - z + 1$	$4z^2 - 4z - 3$
12	Rigid Adiabatic Free Adiabatic	$8z^4 - 4z^3 - 6z^2 - z + 1$	1
13	Rigid Isothermal Rigid Isothermal	$16z^4 - 8z^2 + 1$	$4z^2 - 1$
14	Rigid Adiabatic Rigid Isothermal	$16z^4 - 8z^2 + 1$	$4z^2 + 4z - 3$
15	Rigid Isothermal Rigid Adiabatic	$16z^4 - 8z^2 + 1$	$4z^2 - 4z - 3$
16	Rigid Adiabatic Rigid Adiabatic	$16z^4 - 8z^2 + 1$	1

4. RESULTS

Variation of critical Rayleigh number R_c for different values of Ri, R_{s_1}, R_{s_2}, τ_1 and τ_2 are considered and analysed for all 16 boundary conditions. In all the plots below, the red curves represent the case where both the plates are free, the black curves represent lower free, and upper rigid and the magenta curves represent both rigid. Some of the plots are shown in Figures 2–4. Here, a is the critical wave number corresponding to R.

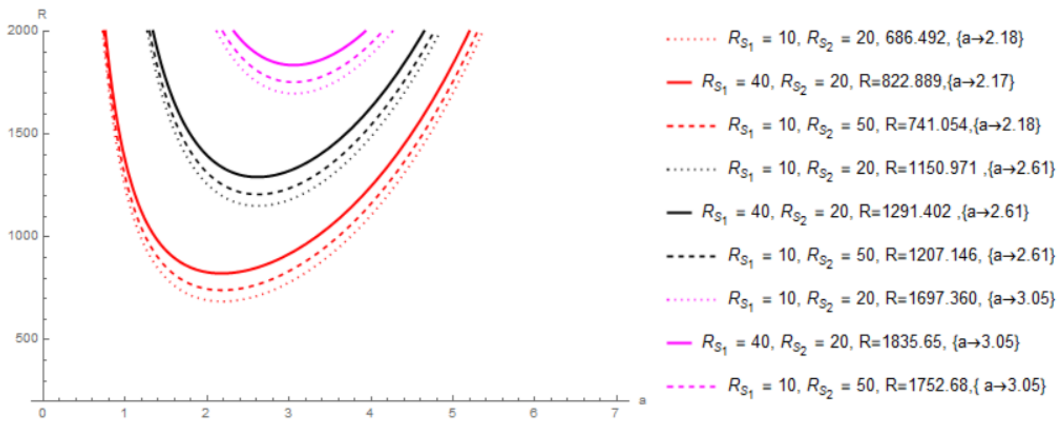

Figure 2: **Plot for R vs a for different values of R_{s_1} and R_{s_2} for isothermal- isothermal ($Ri = 1$, $\tau_1 = 0.2$ and $\tau_2 = 0.5$).**

For the Adiabatic-Adiabatic case, the critical wave number is nearly equal to zero, hence we plot R with respect to any other parameters. Here, we consider R_{s_1}. Analysis of all the parameters along R can be done and the stability of the system can be understood. From Figure 2, we can observe that an increase in R_{s_1} and

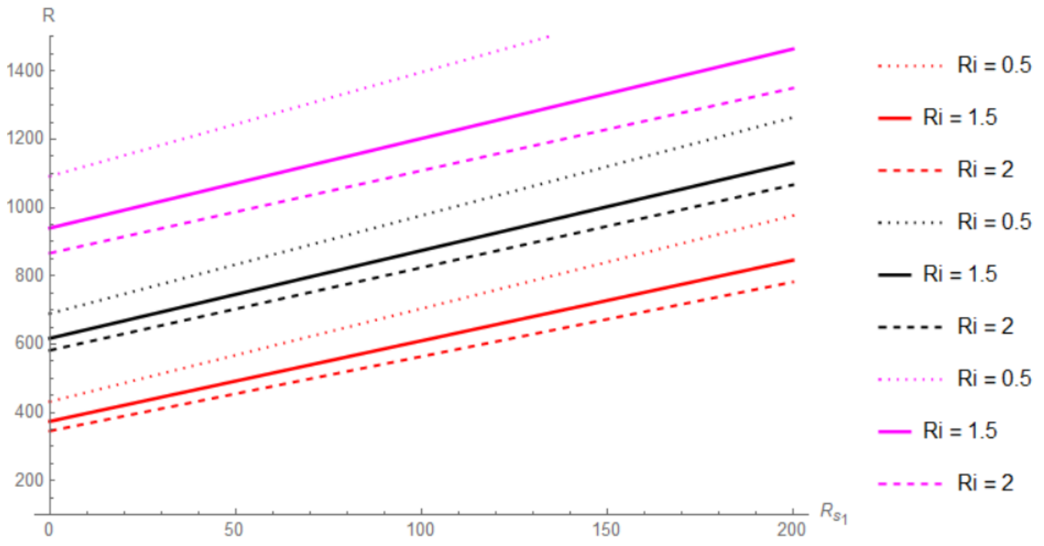

Figure 3: **Plot for R vs R_{s_1} for different values of Ri for adiabatic-adiabatic($Ri = 1$, $R_{s_2} = 20$, $\tau_1 = 0.2$ and $\tau_2 = 0.5$).**

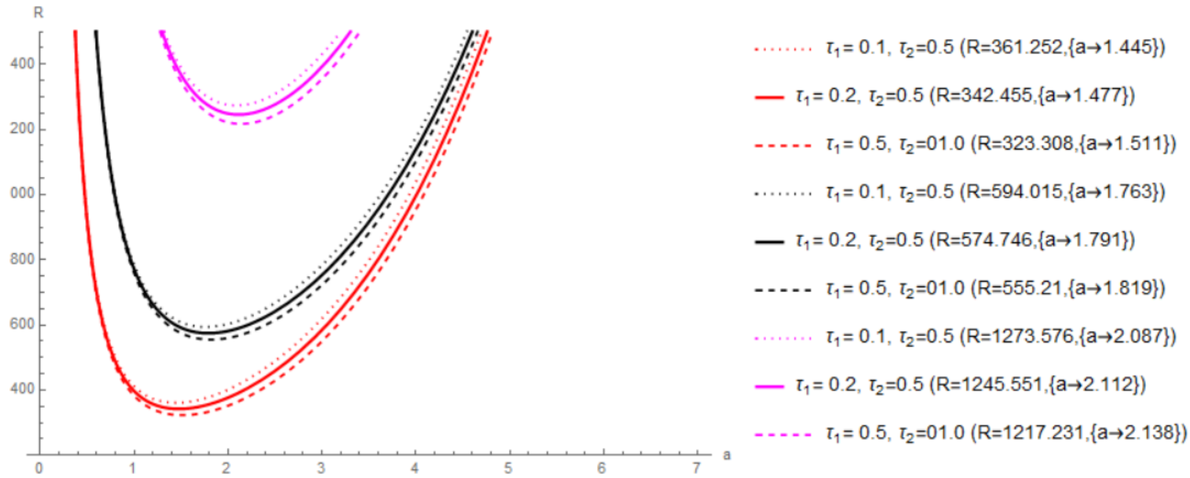

Figure 4: **Plot for R vs a for different values of τ_1 and τ_2 for isothermal–adiabatic($Ri = 1$, $R_{s_1} = 10$ and $R_{s_2} = 20$).**

R_{s_2} increases the critical Rayleigh number which delays the onset of convection and becomes more stable. Since we add solutes from below, the concentration of solutes settles at the bottom without disturbing the system. If the solutes are added from above, then R_{s_1} and R_{s_2} becomes negative and hence decelerates the onset of convection. Whereas, an increase in τ_1 and τ_2 decreases R_c which accelerates the onset of convection leading to the destabilisation of the system. From 3, we can observe that R_c decreases with increase in Ri which says that an increase in internal heat source in the system destabilises the system.

5. CONCLUSION

The linear stability analysis of triple diffusion of the RBC in Newtonian fluids with internal heat generation is investigated theoretically and dimensionless parameters can determine the acceleration or deceleration of the onset of convection for all 16 boundary conditions. Solute Rayleigh number, Internal Rayleigh number and ratios of solute diffusivities help in understanding the stability of this system. An increase in the internal heat source and the ratio of solute diffusivity destabilize the system whereas an increase in solute concentration from the lower plates stabilises the system.

REFERENCES

1. G. Melathil, S. Pranesh, and S. Tarannum, "Effects of magnetic field and internal heat generation on triple diffusive convection in an oldroyd-b liquid," Int. J. Res. Advent Technol, vol. 7, pp. 154–163, 2019.

2. . Poulikakos, "The effect of a third diffusing component on the onset of con- vection in a horizontal porous layer," The Physics of fluids, vol. 28, no. 10, pp. 3172–3174, 1985.

3. N. Rudraiah and D. Vortmeyer, "The influence of permeability and of a third diffusing component upon the onset of convection in a porous medium," International Journal of Heat and Mass Transfer, vol. 25, no. 4, pp. 457–464, 1982.

4. S. Tarannum and S. Pranesh, "Triple diffusive convection in oldroyd-b liq- uid," OSR J. Math, vol. 12, pp. 7–13, 2016.

5. N. Arshika S, S. Tarannum, P. Subbarama et al., "Effect of internal heat source modulations on the onset of triple diffusive convection in viscoelastic liquids," Indian Journal of Engineering and Materials Sciences (IJEMS), vol. 28, no. 5, pp. 509–519, 2021.

6. H. B´enard, Les tourbillons cellulaires dans une nappe liquide propageant de la chaleur par convection, en r´egime permanent. Gauthier-Villars, 1901.