Two-component Magneto-Convection in a porous medium with heat source under realistic boundary

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1. INTRODUCTION & OBJECTIVE

The presence of two or more components with different molecular diffusivity in a gravitational fields develops a new field of convection known as Two-Component Convection. The two-component convection occurs due to the interaction of gravity field with gradience of fluid density. Oceanography is the prime cause of research in two-component convection. Convection in the presence of strong magnetic field is known as magneto-conection. A magneto-convection helps in controlling stability and heat transfer in the system. Many authors have studied magneto-convection under different situations.

In the study by Pranesh, the effect of non-uniform basic concentration gradient on the onset of double diffusive convection in a micropolar fluid layer is studied and linear stability of the system is performed [1]. And in the study of P. G. Siddheshwar and S. Pranesh, the influence of the various micropolar fluid parameters and magnetic field on the onset of stationary convection has been analysed [2]. An Analytical Study of Nonlinear Double-Diffusive Convection in a Porous Medium under Temperature/Gravity Modulation as been studied by P. G. Siddheshwar [3]. Non-linear stability analysis of double-diffusive convection was studied by N. Deepika [4]. A. Hill introduced the heat source into the component convection with porous medium [5].

The objective of this paper is to study two-component magneto-convection in a saturated porous media with heat source in a Newtonian fluid bounded by the realistic boundaries.

The governing equations considered for the study are as follows –

1. Continuity equation

$$\nabla . \mathbf{q} = 0 \qquad (1)$$

2. Conservation of linear momentum

$$\rho_0 \left[\frac{1}{\phi} \frac{\partial t}{\partial \mathbf{q}} + \frac{1}{\phi^2} (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla p + \mu \nabla^2 \mathbf{q} - \rho g \hat{k} - \frac{\bar{\mu}}{k} \mathbf{q} + \mu_m (\mathbf{H} \cdot \nabla) \mathbf{H} \qquad (2)$$

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3. Conservation of Energy

$$A_h \frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)\mathrm{T} = \mathrm{K}\,\nabla^2\mathrm{T} + \mathrm{Q}_1(\mathrm{T} - \mathrm{T}_0) \qquad (3)$$

4. Concentration Equation

$$\phi\,\frac{\partial \mathrm{C}}{\partial \mathrm{t}} + (\mathbf{q}.\nabla)\mathrm{C} = \mathrm{K}_s\nabla^2\mathrm{C} + \mathrm{Q}_2(\mathrm{C} - \mathrm{C}_0) \qquad (4)$$

5. Magnetic induction equation

$$\phi\,\frac{\partial \mathbf{H}}{\partial \mathrm{t}} + (\mathbf{q}.\nabla)\mathbf{H} = (\mathbf{H}.\nabla)\mathbf{q} + \gamma_m\nabla^2\mathbf{H} \qquad (5)$$

6. Magnetic Continuity equation

$$\nabla.\mathbf{H} = 0 \qquad (6)$$

7. Equation of state

$$\rho = \rho_0[1 - \alpha_t\,(\mathrm{T} - \mathrm{T}_0) + \alpha_s(\mathrm{C} - \mathrm{C}_0) \qquad (7)$$

Where the notations meaning remains the same.

And the boundary conditions are taken as–

$$\mathrm{W} = \frac{\partial \mathrm{W}}{\partial \mathrm{Z}} = \mathrm{T} = \mathrm{C} = 0 \ \text{ at } z = -\frac{d}{2} \text{ and } z = \frac{d}{2}$$

The expression for Rayleigh number is obtained as

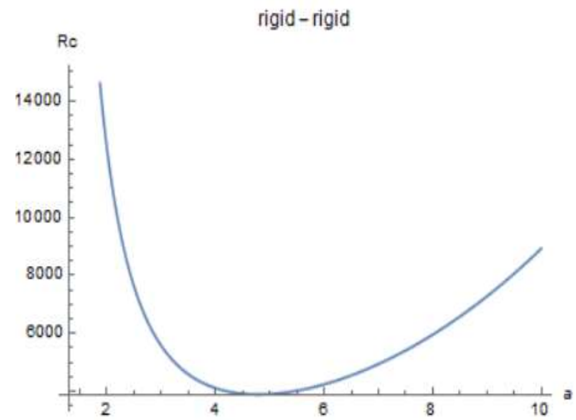
$$\mathrm{Ra} = \frac{[(\mathrm{A_h}\sigma - \mathrm{Ri})\mathrm{I}_4 - \mathrm{I}_5][\mathrm{I}_2 - \left(\frac{\Lambda}{\mathrm{Da}} + \frac{\sigma}{\mathrm{Pr}}\frac{1}{\phi}\right)\mathrm{I}_1 - \frac{\mathrm{Ra_s}\,a^2\mathrm{I}_6}{(\phi\sigma - \mathrm{Ri})\mathrm{I}_7 - \tau\mathrm{I}_8} + \mathrm{Q}\frac{\mathrm{Pr}}{\mathrm{Pm}}\frac{\mathrm{I}_9}{\phi\sigma\mathrm{I}_{10} - {}^{\mathrm{Pr}}/_{\mathrm{Pm}}\,\mathrm{I}_{11}}}{a^2\mathrm{I}_3}$$

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

In this paper, the effect of temperature and concentration-dependent internal energy source, along with magnetic field in a Newtonian fluid over a porous medium is investigated using linear and non-linear stability analysis.

It should be noted that the strength of both source is taken to be the same and moderate values are considered. Ri > 0 and Ri < 0 respectively represent the source and sink. The moderate values of Ra_s are considered and the value of τ is taken as < 1 because heat diffusivity is more compared to solute diffusivity. Because of the presence of suspended particles due to which viscosity increases, the Pr value is taken greater than that of the fluid without suspensions.

The critical Rayleigh number obtained using the Galerkien technique is plotted against wave number for different values of non-dimensional parameters Ri, Ra_s, τ, Q, Pr, Pm, Da, Λ , and ϕ for isothermal cases are obtained.



{3892.14, {a → 4.78718}}

3. CONCLUSION

The rigid-rigid bodies will stabilize the system more, whereas, free-free stabilizes the least and rigid-free lies between rigid-rigid and free-free bodies.

REFERENCES

[1] S. Pranesh and A. K. Narayanappa, "Effect of Non-Uniform Basic Concentration Gradient on the Onset of Double-Diffusive Convection in Micropolar Fluid," *Applied Mathematics*, vol. 3, no. 5, Article ID 19051, pp. 1-8, 2012, doi: 10.4236/am.2012.35064.
[2] P. G. Siddheshwar and S. Pranesh, "Magnetoconvection in a Micropolar Fluid," *International Journal of Engineering Science*, vol. 36, no. 10, pp. 1173-1181, Aug. 1998.
[3] P. G. Siddheshwar, B. S. Bhadauria, and A. Srivastava, "An Analytical Study of Nonlinear Double-Diffusive Convection in a Porous Medium Under Temperature/Gravity Modulation," *International Journal of Engineering Science*, vol. 91, pp. 585-604, Sep. 2011.
[4] N. Deepika, P. A. L. Narayana, and A. A. Hill, "The Nonlinear Stability Analysis of Double-Diffusive Convection with Viscous Dissipation Effect," *Transport in Porous Media*, vol. 150, no. 1, pp. 215-227, 2023, doi: 10.1007/s11242-023-02006-3.
[5] A. A. Hill, "Double-diffusive Convection in a Porous Medium with a Concentration Based Internal Heat Source," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 461, no. 2054, pp. 561-574, 2005, doi: 10.1098/rspa.2004.1328.
[6] M. S. Malashetty and B. S. Biradar, "The Onset of Double Diffusive Convection in a Binary Maxwell Fluid Saturated Porous Layer with Cross Diffusion Effects," *Physics of Fluids*, vol. 23, no. 6, p. 13, 2011, doi: 10.1063/1.3601482.