Triple-Diffusive Convection in a Bi-viscous Bingham Fluid Layer with Temperature Modulation

S. Keerthana1[a] , P. G. Siddheshwar1[a] and Sameena Tarannum2[b]

[a] Centre for Mathematical Needs, Department of Mathematics, CHRIST (Deemed to be University), Bengaluru 560029, India
[b] Department of Professional Studies, CHRIST (Deemed to be University), Bengaluru 560029, India

1. INTRODUCTION & OBJECTIVE

One of the effective ways to control convection is by maintaining the non-uniform temperature gradient, which solely depends on space. The thermal modulation problem, which involves solving the energy equation under appropriate time-dependent boundary conditions, arises because transient heating or cooling at boundaries is the cause of non-uniform temperature gradients in many practical settings. As a result, the basic temperature profile is explicitly dependent on time and position. By appropriately adjusting the modulation parameters, namely the amplitude and frequency, these basic temperature profiles can be used as an efficient means of controlling convective flow. There may be a noticeable increase in heat, mass, or momentum. It has widespread applications in solidifying metallic alloys in crystal growth to control the quality and structure of the solid and material processing to achieve higher efficiency. Venezian [1] was the first to examine the effect of modulation on the onset of thermal-convection using perturbation method and concluded that the shift in the critical Rayleigh number is calculated as a function of frequency, and he observed that it is feasible to progress or postpone the onset of convection by time modulation of the wall temperatures. The authors [2-13] have researched temperature modulation in double and triple diffusive convection for different scenarios.

In this study, we consider a bi-viscous Bingham fluid, a non-Newtonian fluid whose flow is not linear and exhibits yield-stress and plastic viscosity simultaneously. This fluid involves a Newtonian fluid with deformable, suspended particles that enhance viscosity and alter the stress's nature. The suspended particles impart a little more rigidity to the fluid, making it respond slowly to any force applied to it. This gives rise to the concept of a yield-stress. When the yield-stress approaches infinity, the BVBF can be considered a Newtonian fluid. The shear rate of a fluid at rest with no external forces has a zero-shear rate; once the shear rate reaches the threshold, it increases linearly with the strain rate. Because of the imposed shear rate, the fluid viscosity drops and begins to flow. This form of fluid flow is used in various industrial and everyday applications, such as drilling operations and well completion technologies. Some of the materials that manifest almost all this model's characteristics are polymeric-gels, emulsions, mud, heavy-oil and foams with nonzero yield-stress, and are often used in oil-recovery or fracking-applications. Due to the unique property of BVBF, several authors [14-22] have extended earlier works on BVBF for different conditions. Very recently, convection problems were studied for other situations by Turan et al. [23], Mahanthesh et al. [24], and Nandal et al. [25].

Therefore, the foremost objective of this paper is to study the effect of imposed temperature modulation on the stability of convection flow and heat and mass transfers in a bi-viscous Bingham fluid layer by considering free-free velocity boundaries.

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

A theoretical study of thermo-convective instability in a bi-viscous Bingham fluid layer is carried out when the boundary temperatures vary with time in a sinusoidal manner. By performing a linear and weakly non-linear stability analysis, we obtain the expressions for the correction Rayleigh number for the onset using the perturbation method and the Nusselt and Sherwood numbers as a function of the amplitude of convection which is governed by a non-autonomous Ginzburg Landau equation derived for the stationary mode of convection. Furthermore, the temperature modulation can either stabilize or destabilize depending on the value of the modulation frequency. Additionally, we see that average Nusselt and Sherwood numbers increase with an increase in bi-viscous Bingham fluid parameter, solutal Rayleigh numbers, and modulation frequency. In contrast, they decrease with an increase in solutal diffusivity rate and modulation amplitude. The heat and mass transfers in the interval $[0, \pi]$ decrease with an increase in the phase angle, φ for in-phase, and they increase in the interval $[\pi, 2\pi]$ for out-of-phase. The values of heat- and mass-transfers have a maximum at $\varphi \approx 2\pi$ and a minimum at $\varphi \approx \pi$.

REFERENCES

1. G. Venezian, “Effects of modulation on the onset of thermal convection,” *J. Fluid Mech*. **35**, pp. 243–254, 1969.
2. B. S. Bhadauria, P. G. Siddheshwar and Om P. Suthar, “Nonlinear Thermal Instability in a Rotating Viscous Fluid Layer Under Temperature/Gravity Modulation,” *J. Heat Transf.* **134,** pp. 102502-1, 2012.
3. P. G. Siddheshwar and S. Pranesh, “Effect of temperature/gravity modulation on the onset of magneto-convection in electrically conducting fluids with internal angular momentum,” *J. Magn. Magn. Mater*. **219**, pp. 153-162, 2000.
4. B. S. Bhadauria, I. Hashim, and P. G. Siddheshwar, “Effects of Time-Periodic Thermal Boundary Conditions and Internal Heating on Heat Transport in a Porous Medium,” *Transp. Porous Media.* **97**, pp. 185-200, 2013.
5. K. Thriveni and S. Pranesh, “Linear and Non-Linear Analysis of Solute-Convection Modulation in A Couple Stress Fluid under Temperature Modulation,” *Int. J. Appl. Eng. Res.* **13**, pp. 16984-16994, 2018.
6. B. S. Bhadauria, P. G. Siddheshwar, A. K. Singh, and Vinod K. Gupta, “A Local Nonlinear Stability Analysis of Modulated Double Diffusive Stationary Convection in a Couple Stress Liquid,” *J. Appl. Fluid Mech.* **9**, pp. 1255-1264, 2016.
7. P. G. Siddheshwar and Annamma Abraham, “Rayleigh-Benard Convection in a Dielectric Liquid : Imposed Time-Periodic Boundary Temperatures,” *CMJ.* **1**, pp. 105-121, 2009.
8. S. Pranesh and Sangeetha Georgea, “Effect of imposed time periodic boundary temperature on the onset of Rayleigh-Bénard convection in a dielectric couple stress fluid,” *Int. J. Appl. Math. Comput. Sci*. **5**, pp. 1-13, 2014.
9. I. S. Shivakumara, Jinho Lee, M. S. Malashetty, and S. Sureshkumar, “Effect of Thermal Modulation on the Onset of Convection in Walters B Viscoelastic Fluid-Saturated Porous Medium,” *Transp. Porous Media.* **87**, pp. 291-307, 2011.
10. S. Pranesh and R. V. Kiran, “Rayleigh-Bénard Chandrasekhar Convection in An Electrically Conducting Fluid using Maxwell-Cattaneo Law with Temperature Modulation of the Boundaries,” *Int. J. Eng. Res. Technol.* **4**, pp. 2278-0181, 2015.
11. A. Neha Aanam and S. Pranesh, “Synchronous and Asynchronous Temperature Modulation of Rayleigh-Bénard Chandrasekhar Convection in Couple Stress Fluid using Ginzburg-Landau Model,” *Int. J. Appl. Eng. Res.* **14**, pp. 523-533, 2019.
12. T. Sameena and S. Pranesh, “Rayleigh-Bénard Chandrasekhar Convection in An Electrically Conducting Fluid using Maxwell-Cattaneo Law with Temperature Modulation of the Boundaries,” *Int. J. Eng. Res. Technol.* **7**, pp. 645-652, 2018.

13. S. Pranesh, T. Sameena and J. Meghana, "Effects of sinusoidal and non-sinusoidal temperature modulation in a triple diffusive convection," *TWMS J. of Appl. & Eng. Math.* **11**, pp. 1207-1220, 2021.
14. A. Lenci and V. Di Federico, "A channel model for Bi-viscous fluid flow in fractures," *Transp. Porous Media.* **134**, pp. 97-116, 2020.
15. I. A. Frigaard, S. D. Howison, and I. J. Sobey, "On the stability of Poiseuille flow of a Bingham fluid," *J. Fluid Mech.* **263**, pp. 133-150, 1994.
16. M. Nakamura and T. Sawada, "Numerical study on the laminar pulsatile flow of slurries," *J. Nonnewton. Fluid Mech.* **22**, pp. 191–206, 1987.
17. M. Nakamura and T. Sawada, "Numerical study on the flow of a non-Newtonian fluid through an axisymmetric stenosis," *J. Biomech. Eng*. **110**, pp. 137-143, 1988.
18. A. Lavrov, "Modeling flow of a Biviscous fluid from borehole into rock fracture," *J. Appl. Mech*. **73**, pp. 171-173, 2006.
19. O. Turan, N. Chakraborty, and R. J. Poole, "Laminar natural convection of Bingham fluids in a square enclosure with differentially heated side walls," *J. Nonnewton. Fluid Mech*. **165**, pp. 901-913, 2010.
20. L. Talon and A. Hansen, "Effective rheology of Bi-viscous non-Newtonian fluids in porous media," *Front. Phys***. 7**, pp. 1-11, 2020.
21. O. Turan, S. Yigita, and N. Chakraborty, "Critical condition for Rayleigh-Bénard convection of Bingham fluids in rectangular enclosures," *Int. Commun. Heat Mass Transf.* **86**, pp. 117-125, 2017.
22. A. Raheel, A. Nasir, U. K. Sami, C. Ali, and T. Iskander, "Heat and mass transfer characteristics in flow of bi-viscosity fluid through a curved channel with contracting and expanding walls: A finite difference approach," *Adv. Mech. Eng.* **12**, pp. 1-6, 2020.
23. A. Lenci and V. Di Federico, "A channel model for Bi-viscous fluid flow in fractures," *Transp. Porous Media.* **134**, pp. 97-116, 2020.
24. B. Mahanthesh, M. Shruthy and V. Kuppalapalle, " Study of Multilayer Flow of a Bi-Viscous Bingham Fluid Sandwiched Between Hybrid Nanofluid in a Vertical Slab with Nonlinear Boussinesq Approximation," *Phys. Fluids.* **34**, pp. 122006, 2022.
25. R. Nandal and P. G. Siddheshwar, "Thermorheological effect on Rayleigh–Bénard magnetoconvection in a biviscous Bingham fluid with rough boundary condition on velocity and Robin boundary condition on temperature," *Heat Transf*. **52**, pp. 2943-2974, 2023.