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Dynamic Behavior of Piezoelectric Material Due to Electromechanical Coupling through Non-ordinary State-based Peridynamic Theory

Subrata Mondal*
Department of Mathematics, School of Sciences, National Institute of Technology Andhra Pradesh, India,
*E-mail: pf122102@student.nitandhra.ac.in; subratamath01@gmail.com

ABSTRACT

Research aim: In this study, the dynamic behaviour of piezoelectric materials has been investigated within the framework of peridynamic theory. It introduces and applies peridynamics in a coupled field setting to address electromechanical problems. An analysis of plane wave propagation in electromechanical systems has been conducted for the first time within the framework of non-ordinary state-based peridynamic. The dispersion relation for longitudinal and transverse wave is determined by analyzing the harmonic solutions. Validation was achieved both analytically and numerically by ensuring classical correspondence in the limit of the nonlocality parameters approach zero. The effective parameters such as the size of the horizon, different nonlocality functions, and shape parameters of nonlocality functions significantly affect the dispersion curves. The influence of electromechanical coupling parameters on frequency, phase velocity, and group velocity has been shown to have significant effects. Understanding their behaviour and properties from a wave and vibrational perspective is essential for designing efficient and effective piezoelectric devices. Their ability to convert mechanical energy into electrical energy and vice versa makes them indispensable in sensors, actuators, transducers, energy harvesters, and medical imaging devices.

Literature survey: Peridynamics (PD) is a theoretical framework that employs integrodifferential equations as an alternative to the traditional partial differential equations used in classical continuum mechanics. Silling [1] introduced the mathematical formulation of PD in 2000. The propagation of waves has been investigated with the aid of both state-based and bond-based PD in various works [2, 3, 4]. Piezoelectric materials have attracted attention as new smart materials. As a result, novel composite structures are now being rapidly developed with increasing advancements in material science and engineering. Recently, the formulation for electromechanical coupling in PD has been developed. A few researchers [5, 6, 7] have explored the effects of electromechanical coupling in peridynamic media. However, the dynamic behaviour of piezoelectricity within the framework of peridynamic theory has yet to be addressed in the literature. Through theoretical modelling and numerical simulations, this article aims to uncover the intricate interplay between nonlocal behaviours and wave propagation on piezoelectric material through peridynamic theory.

Problem formulation and Solutions: The equations of equilibrium of non-ordinary state-based linear piezoelectricity and the equation of the electrostatic field are as follows:

$$\begin{aligned} &\int_{\mathcal{H}} \left\{\underline{\mathbf{T}}[\mathbf{x}]\langle\xi\rangle - \underline{\mathbf{T}}[\mathbf{x}']\langle\xi\rangle\right\} dV + \mathbf{b}(\mathbf{x}) = \mathbf{0} \\ &\int_{\mathcal{H}} \left\{\underline{\mathbf{d}}[\mathbf{x}]\langle\xi\rangle - \underline{\mathbf{d}}[\mathbf{x}']\langle\xi\rangle\right\} dV = 0 \end{aligned} \qquad (1)$$

Hence, the expressions for the PD vector state and for the PD electric displacement scalar state can be written as:

$$\begin{aligned} \underline{\mathbf{T}}[\mathbf{x}]\langle\xi\rangle &= \omega\langle\xi\rangle \boldsymbol{\sigma} \mathbf{K}^{-1}(\mathbf{x})\xi \\ \underline{\mathbf{d}}[\mathbf{x}]\langle\xi\rangle &= \omega\langle\xi\rangle \mathbf{D}^{T}(\mathbf{K}^{-1}(\mathbf{x})\xi) \end{aligned} \qquad (2)$$

Where, the shape tensor $\mathbf{K}$ is defined in [5] and $\omega\langle\xi\rangle$ is the weight function defined in [3].

The nonlocal PD Stress and Electric displacement is defined as:

$$\begin{aligned} \boldsymbol{\sigma} &= C\boldsymbol{\varepsilon} - e\boldsymbol{E} \\ \boldsymbol{D} &= e^{T}\boldsymbol{\varepsilon} + \kappa\boldsymbol{E} \end{aligned} \qquad (3)$$

The nonlocal PD strain deformation gradient and the nonlocal PD electric potential gradient as $\boldsymbol{\varepsilon} = \frac{\mathbf{G}_{\underline{\mathbf{U}}}+\mathbf{G}_{\underline{\mathbf{U}}}^{T}}{2}$ and $\mathbf{E} = -\mathbf{G}_{\underline{\Phi}}(\mathbf{x})$, where $\mathbf{G}_{\underline{\mathbf{U}}}(\mathbf{x})$ and $\mathbf{G}_{\underline{\Phi}}(\mathbf{x})$ are defined in [5]. Also, $\boldsymbol{\sigma}$, $\boldsymbol{D}$ are the nonlocal PD stress and nonlocal PD electric displacement, respectively. Also, the elastic stiffness tensor, electric tensor and dielectric tensor are denoted by C, e, and κ, respectively.
Finally, substituting the values of displacement components and electric potentials, in the constitutive relations (2) and then in the equilibrium equations (1), a system of linear homogeneous equations can be obtained as

$$\mathbf{PA} = \mathbf{O}, \tag{9}$$

For a non-trivial solution, the associated determinant of the matrix has to vanish, i.e,

$$\mathbf{Det(P)=0} \tag{10}$$

where **P** is the coefficient matrix. Eq. (10) is the dispersion relations of plane wave propagation in piezoelectric structures within the framework of Peridynamic theory.
The numerical computations and discussions of frequency, phase velocity, and group velocity from dispersion relations for the propagation of the plane wave in peridynamic media have been performed to validate the theoretical findings.
Significant conclusions: In conclusion, the study of the size of the horizon, the number of points inside the horizon, and the shape of the influence function significantly influences the dispersion curves of plane waves in the piezoelectric material. The presence of nonlocal effects further enhances the complexity of wave propagation by introducing long-range interactions and spatial dependencies. Both longitudinal and transverse waves are dispersive due to the nonlocal characteristics of peridynamic media, which are influenced by the nonlocal length parameter (horizon size) and nonlocality functions. It provides insight into whether higher-frequency waves travel faster or slower than lower-frequency waves. The electromechanical coupling parameters have a significant impact on frequency, phase velocity, and group velocity. Understanding the behaviour and properties of electromechanical coupling parameters from a wave and vibrational perspective is essential for designing efficient and effective piezoelectric devices. Their ability to convert mechanical energy into electrical energy and vice versa makes these parameters indispensable in sensors, actuators, transducers, energy harvesters, and medical imaging devices.
Keywords: Non-ordinary state-based peridynamics, Electromechanical Coupling, Piezoelectricity, Wave dispersion, multi-physics.

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