PAPER FOR THE YOUNG SCIENTIST AWARD

Stability Analysis of Rotating Magnetoconvection: Anisotropic Thermal Diffusivity Effect

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ABSTRACT

The stability of horizontal fluid planar layer rotating about its vertical axis, permeated by the horizontal homogeneous magnetic field and heated from below and cooled from above, with the influence of anisotropic thermal diffusivity is studied. The linear stability analysis is investigated near the onset of convection with the normal mode analysis. The stratification anisotropy (SA) parameter, α, which is the ratio of horizontal and vertical thermal diffusivities, plays a key role in deciding the boundaries between stationary cross (SC), oblique (SO) and parallel (P) modes. The weakly nonlinear analysis of the stationary convective motion in the vicinity of onset of convection is studied using multiple scale analysis and the two-dimensional anisotropic Landau-Ginzburg (ALG) equation with cubic nonlinearity has been derived. In the case of high rotation, heat transfer rate (Nu) gets decreased from atmospheric ($\alpha < 1$) to oceanic ($\alpha > 1$) SA types. The domain Eckhaus instability is obtained using the spatio-temporal ALG equation and it is observed that the region decreases with increasing α.

LITERATURE SURVEY

In the geophysical and astrophysical models, the rotation rate and the magnetic field play a key role to understand the convective flow behavior in the Earth's outer core regions and the outer layer of the Sun. Such models are observed in Rotating Magnetoconvection (RMC). Thus, the RMC models along with the Boussinesq approximation have been studied by Chandrasekhar (1961), Roberts and Jones (2000), Šoltis and Brestenský (2010), Rameshwar et al. (2023). They have studied the linear stability of the horizontal plane layer heated from below and cooled from above, rotating about a vertical axis and permeated by a horizontal homogeneous magnetic field using separable solutions near the onset of convection. However, the anisotropic thermal diffusivity effects were not included, although the rising importance of the same in the directional heat transport in industrial and natural processes. The heat transport inside the Earth's core get affected strongly by the anisotropic thermal diffusion (Braginsky and Meytlis (1990) and Phillips and Ivers (2000)). Donald and Roberts (2004) investigated the influence of anisotropic thermal diffusive coefficient (κ) in magnetoconvection. This problem is extended by Šoltis and Brestenský (2010) considering the thermal and viscous diffusion coefficients as anisotropic. More results can be extracted if the weakly nonlinear analysis is performed on the RMC along with stationary convection. Hence an attempt is made in the present study to analyze this RMC in planar layer near the onset of convection, using the normal mode method for the linear stability analysis and the multiple scale analysis (Pesch and Kramer (1986)) for weakly nonlinear analysis. The linear stability analysis of the derived ALG equation is performed to investigate the secondary instabilities, such as, Eckhaus instability.

PROBLEM STATEMENT

The electrically and thermally conducting fluid is assumed to be kept in between the two horizontal infinite planar layers. This cartesian coordinate (x, y, z) configuration with corresponding unit vectors ($\hat{x}$, $\hat{y}$, $\hat{z}$) is considered to be kept under the homogeneous horizontal magnetic field and rotating about its vertical axis. The fluid is assumed to be heated from below and cooled from above. In geophysical applications the material property, κ, is assumed to be anisotropic and the corresponding perturbed non-dimensional governing equations are considered as below (Šoltis and Brestenský (2010), Nayak et al. (2024)):

$$R_o[\boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}] + \hat{\boldsymbol{z}}\times\boldsymbol{u} = -\nabla P + \Lambda(\boldsymbol{b}\cdot\nabla)\boldsymbol{b} + \Lambda\boldsymbol{b}_y + R\theta\hat{\boldsymbol{z}} + E\nabla^2\boldsymbol{u}\,, \quad (1)$$

$$\boldsymbol{b}_t = \nabla\times(\boldsymbol{u}\times\boldsymbol{b}) + \nabla\times(\boldsymbol{u}\times\hat{\boldsymbol{y}}) + \nabla^2\boldsymbol{b}, \quad (2)$$

$$q_z^{-1}[\theta_t + (\boldsymbol{u}\cdot\nabla)\theta] = \hat{\boldsymbol{z}}\cdot\boldsymbol{u} + \nabla_\alpha^2\theta, \quad (3)$$

$$\nabla\cdot\boldsymbol{u} = 0,\ \nabla\cdot\boldsymbol{b} = 0, \quad (4\text{ a, b})$$

and $w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial \omega}{\partial z} = \theta = b = \frac{\partial j}{\partial z} = 0$ at $z = \pm\frac{1}{2}$

where $\boldsymbol{u}$ (= u, v, w) denote the velocity field vector; b, ω and j denote the z-component of magnetic field $\boldsymbol{b}$ = (b_x, b_y, b), vorticity and current density, respectively, θ and P denote the temperature and modified pressure, respectively, and t is the time. The non-dimensional parameters are given by R_0 = Modified Rossby Number, Λ = Elsasser Number, R = Modified Rayleigh Number, E = Ekman Number and q_z = Roberts Number. It can be noted that $\nabla_\alpha^2\theta$ in the heat equation takes care about the anisotropic thermal diffusivity.

SOLUTION METHODOLOGY

The z-component of the curl and double curl of the Navier-Stokes equation (1), the curl of the induction equation (2) and induction equation (2) itself and the equation of heat conduction (3) gave twelfth order nonhomogeneous ordinary differential equation. The linear stability analysis has been carried out with normal mode analysis (Filippi et al. (2019)). The critical R and critical wavenumber have been derived for the case of SC, SO, and P rolls with different values of E and Λ. The weakly nonlinear stability analysis has been studied with multiple scale analysis (Newell and Whitehead (1969)) and ALG equation is derived. The heat transfer rate has been calculated. Eckhaus instability region has been shown for the different values of E, Λ, R_o and q_z.

CONCLUSIONS

It is observed that the range of SC, SO and P modes strongly depends on α. The stratification atmospheric (Sa) anisotropy with $\alpha < 1$, facilitates the convection by decreasing the R_{sc} and on the other hand in the case of stratification oceanic (So) anisotropy with $\alpha > 1$ impedes the onset of convection by increasing the R_{sc}. The *Nu* decreases from atmospheric ($\alpha < 1$) to oceanic ($\alpha > 1$) SA type for fixed E, Λ and R. The exhibited domain for the secondary instabilities strongly depends on α.

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