Numerical Treatment of Singly Perturbed Differential-Difference Equations

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Abstract

The main reasons for the choice of singularly perturbed differential-difference equation models are: (i) they have a richer mathematical framework when compared with ordinary differential equations for the analysis of bio-system dynamics and (ii) they display better consistency with the nature of the underlying processes and predictive results. The class of differential-difference equations which have characteristics of both classes, i.e. delay/advance and singularly perturbed behaviour is known as singularly perturbed differential-difference equations. The expression “positive shift” and “negative shift” are also used for “advance” and “delay” respectively. In general, an ordinary differential equation in which the highest order derivative is multiplied by a small positive parameter and containing at least one delay/advance is known as singularly perturbed differential-difference equation. Singularly perturbed differential difference equations arise in the modelling of various practical phenomena in bioscience, engineering, control theory, such as in variational problems in control theory, in describing the human pupil-light reflex, in a variety of models for physiological processes or diseases and first exit time problems in the modelling of the determination of expected time for the generation of action potential in nerve cells by random synaptic inputs in dendrites. In applications, the future behaviours of many phenomena are assumed to be described by the solution of an ordinary differential equation. Implicitly this assumption means that the future behaviour is uniquely determined by the present and independent of the past. However, in differential-difference equations, the past exerts its influence in a significant manner upon the future. Many models are better represented by functional differential equations in general and differential-difference equations in particular, than ordinary differential...
equations. In solving singularly perturbed differential-difference equations, perturbation methods such as WKB method together with matched asymptotic expansions are used extensively. These asymptotic expansions of solutions require skill, insight and experimentation. Further, matching of the coefficients of the inner and outer regions solution expansions is also a demanding process. Hence, researchers started using numerical methods. The numerical methods developed for regular perturbation problems turn out inapplicable for solving singular perturbation problems as the solution profile in this case depend upon the value of the singular perturbation parameter. Errors in the numerical solutions depend upon the distribution of the mesh points and become small only when the effective mesh size in the layer is much less than the value of the perturbation parameter. In general, the efficiency of a numerical method is determined by its accuracy, simplicity in computing the numerical solution and its sensitivity to the parameters of the given problem. Hence, there is a need to develop numerical methods satisfying these requirements. This motivates the researchers to develop simple, non asymptotic, easy to use and efficient numerical methods which are readily adaptable for computer implementation with a modest amount of problem preparation. In response to this, we have proposed and illustrated some simple, easy and efficient numerical methods for finding the numerical solution of singularly perturbed differential-difference equations involving negative shift; and both negative and positive shifts. All the methods presented have been implemented on several model examples for different values of the delay, advance and perturbation parameters. The numerical solutions are tabulated and compared with the exact solutions. It is observed that the present methods approximate the exact solution very well. The beauty of these methods is that they do not depend on asymptotic expansions and matching of the coefficients. Further, they do not require very fine mesh size. Above all, these methods are conceptually simple, easy to use and are readily adaptable for computer implementation with a modest amount of problem preparation.