Dynamic rupture propagation at non-planar interface using boundary integral equation

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1. INTRODUCTION & OBJECTIVE

The effect of fault geometry on earthquake mechanics is one of the fundamental questions in seismology. For this, a considerable number of works has been conducted analytically and numerically to understand the effect of geometrical complexities and roughness on the dynamic rupture of the earth model (Rice et al., 2005; Fang and Dunham, 2013; Dunham et al., 2011; Sathiakumar and Barbot, 2021). It involves fully dynamic earthquake cycle simulations on non-planar faults, which require much larger computational costs than those with planar faults. From past studies, it is evident that the conventional traction boundary integral element method (BIEM) is one of the popular methods which gives the analytical solution of the stress field based on the convolution history of displacement along the fault. However, the traction boundary integral equations (BIEs) are hypersingular (Aki and Richards, 2002) and possess difficulty in immediate numerical implementation. This problem can be solved by regularisation and adopted by many researchers (Sato et al., 2020; Romanet et al., 2020; Tada and Yamashita, 1997) in the past. Generally, in the numerical solution procedure, BIEM in the space-time domain is converted into a wavenumber domain using Fast Fourier Transformation (FFT). Apart from this, the analytical nature of BIEM requires consideration of a simple medium during simulation. Also, the spectral form of BIEM requires equispaced data, which is a drawback in the non-planar faults. Thus, there is a need for more effective methods of simulating the earthquake cycle using fully dynamic BIEs that may be applied to non-planar faults. It is worth mentioning that some researchers have used a different approach, which yields weakly singular displacement BIEs for planar or straight-crack problems (Kostrov, 1975; Das, 1980). These displacement BIEs are equivalent to the traction BIEs in reverse form. When applied to an interface between heterogeneous elastic half-spaces, these displacement BIEs have yielded simple and closed-form convolution kernels (Ranjith 2015; Ranjith 2022). It is clear from past work that displacement BIEs can smoothly handle the material heterogeneity in earthquake simulation, which is often difficult for traction BIEs. Displacement BIEs of this kind have not been utilised to analyse non-planar fracture situations until now.

Here, we present a preliminary formulation from our ongoing work investigating displacement BIEs applicability for non-planar faults. This work aims to provide a thorough investigation to ascertain the effects of non-planar fault geometry on the interfacial shear stress, slip velocity, and slip of similar elastic half-spaces. The main topics of our discussion are two fault modes: opening and in-plane shear. In this work, non-planar 2-D fractures are analysed dynamically using displacement

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BIEs based on the Kostrov formulation are weakly singular or non-hypersingular. We expanded the slip-traction relation of a non-planar crack to the first order to determine the deviation from a planar fault geometry. We are presenting a general mathematical formulation independent of specific geometry and normal and shear stress distribution in the regime of a small fault slope.

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

Here, we consider a three-dimensional elastodynamic problem considering a linear, elastic, infinite body bounded by volume v and surrounded by surface s. The final representation theorem for a homogeneous half-space (Das, 1980) computes the displacement $U_n(\vec{Y}, t)$ in the space and time t can be written as

$$U_n(\vec{Y},t) = \int_0^t dt \int_s n_j g_{ni}(\vec{Y} - \vec{X}, t - \text{r})\tau_{ij}(\vec{X},t)\, ds \tag{1}$$

where, n_j is the normal vector in the j^{th} direction, g_{ni} refers to a fundamental solution in the n^{th} direction due to applied force in the i^{th} direction, τ_{ij} stands for traction components acts at $(\vec{X}, t)$ due to the applied impulsive load at $(\vec{Y}, \text{r})$, consisting of elasticity modulus C_{ijpq} and derivative of different displacement components in different directions $U_{p,q}$.

In the two-dimensional dynamic problem, a parallel representation theorem is used. We put both $\vec{X}$ = (x_1, 0) and $\vec{Y}$ = (y_1, 0) on the plane $x_2 = 0$ with load applied at initial time $\text{r} = 0$ and we get

$$U_n(y_1,t) = \int_0^t dt \int_s n_j g_{ni}(y_1 - x_1, t)\tau_{ij}(x_1)\, ds \tag{2}$$

For homogeneous half spaces, accounting for the symmetry of displacement components, we obtain reduced representation relations in terms of slip α and traction components, as

$$\alpha_n(y_1,t) = U_n^{upper}(y_1,t) - U_n^{lower}(y_1,t) = 2\int_0^t dt \int_s n_j g_{ni}(y_1 - x_1, t)\tau_{ij}(x_1)\, ds \tag{3}$$

By incorporating a small slope assumption, the integrals along the fault are computed along the x_1 axis only. This is due to the fact that if the slope derivative is small, the variable of the integral will be changed, as

$$\int_s f(x_1)\, ds = \int_{-\infty}^{\infty} f(x_1)\sqrt{1 + h'^2(x_1)}dx_1 \simeq \int_{\infty}^{-\infty} f(x_1)\, dx_1 \tag{4}$$

We study the two-dimensional geometry of a plane fracture, the non-zero components of the initially applied stress are τ_{11}, τ_{12}, and τ_{22}. $\alpha_1(y_l)$ and $\alpha_2(y_1)$ give opening and in-plane shear components of the displacement discontinuity, which are both in the plane (x_l-x_2). We can determine the integral equations to find α_1 and α_2, putting n = 1, 2 on x_2 = 0 using equation (3) and small slope approximation.

$$\alpha_n(y_1,t) = 2\int_0^t dt \int_{-\infty}^{\infty} n_j g_{ni}(y_1 - x_1, t)\tau_{ij}(x_1)\, dx_1 \tag{5}$$

The expression for Green's function can be found in Eringen and Suhubi (2013)

$$g_{11}(x_1 - y_1, x_2 - y_2) = Re\frac{1}{i\pi\mu|x_1 - y_1|}\left[-\frac{2\gamma_1{}^2(c_2{}^{-2} - \gamma_1{}^2)^{1/2}}{R(\gamma_1{}^2)\Delta_1'} - \frac{(c_2{}^{-2} - 2\gamma_2{}^2)(c_2{}^{-2} - \gamma_2{}^2)^{1/2}}{R(\gamma_2{}^2)\Delta_2'}\right] \tag{6}$$

$$g_{22}(x_1-y_1,x_2-y_2)=Re\,\frac{1}{i\pi\mu|x_1-y_1|}\left[-\frac{\left(c_1{}^{-2}-\gamma_1{}^2\right)^{1/2}\left(c_2{}^{-2}-2\gamma_1{}^2\right)}{R(\gamma_1{}^2)\Delta_1^{'}}-\frac{2\gamma_2{}^2\left(c_1{}^{-2}-\gamma_2{}^2\right)^{1/2}}{R(\gamma_2{}^2)\Delta_2^{'}}\right] \tag{7}$$

where,

$$\Delta_\alpha^{'}=-(x_1-y_1)+\frac{(x_2-y_2)\gamma_\alpha}{(c_\alpha{}^{-2}-\gamma_\alpha{}^2)^{1/2}};\ \gamma_\alpha=\frac{(x_1-y_1)t+i(x_2-y_2)\left[t^2-\left(\frac{r}{c_\alpha}\right)^2\right]^{1/2}}{r^2}$$

$$R(\gamma_\alpha{}^2)=(c_2{}^{-2}-2\gamma_\alpha{}^2)^2+4\gamma_\alpha{}^2(c_1{}^{-2}-\gamma_\alpha{}^2)^{1/2}(c_2{}^{-2}-\gamma_\alpha{}^2)^{1/2};\ r=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$$

The displacement BIEs can be divided into the 0^{th} and 1^{st}-order solutions of the fault's slip due to traction distribution for different slip modes using the small slope approximation. The slip can be divided into 0^{th}, 1^{st}, and higher-order terms in the wave number (k) domain as follows

$$\underbrace{\alpha(k,t)}_{\text{Full solution}}=\underbrace{\alpha^0(k,t)}_{0^{\text{th}}\text{ order}}+\underbrace{\alpha^1(k,t)}_{1^{\text{st}}\text{ order}}+\underbrace{\cdots}_{\substack{\text{Higher}\\\text{orders}}} \tag{8}$$

We are currently developing simulations to understand the physics of non-planar faults. We conducted mathematical formulation for non-planar faults using displacement BIEs based on the Kostrov formulation. We perturb the slip-traction relationship into the zeroth and first-order term to show deviation from planar fault geometry. We also assumed the small slope geometry and implemented it in the BIEs and Green's function to assess the effect of nonplanarity. This study will help us to understand the effect of fault geometry on the slip distribution, velocity and interfacial stress along the fault trace.

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