Effect of gravity modulation on the onset of Rayleigh-Bénard convection in a ferrofluid in the presence of nano particles

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1. ABSTRACT

The study investigates the impact of sinusoidal (sine waves) and non-sinusoidal (square wave, triangular wave and sawtooth wave) gravity modulation on Rayleigh-Bénard convection in a ferrofluid containing nanoparticles through both linear and non-linear stability analyses. This study examines the effects of various dimensionless parameters on the onset of convection and heat transfer. The findings indicate that magnetization parameters destabilize the system and increases heat transfer. Moreover, the square wave gravity modulation is found to facilitate greater heat transport compared to other forms of gravity modulation.

2. RESEARCH AIM

The research aim includes assessing system stability using linear analysis, evaluating heat transfer and understanding how different modulations influence heat transfer, examining how various dimensionless parameters including magnetization and ferromagnetic parameters affect the onset of convection and heat transfer, comparing the effectiveness of sinusoidal versus non-sinusoidal gravity modulation in facilitating heat transfer and elucidating the effects of nanoparticle presence on the stability and efficiency of heat transfer in ferrofluids undergoing Rayleigh-Bénard convection.

3. BRIEF LITERATURE SURVEY

The need of heat transfer fluids in industrial applications has received much importance. The commonly used heat transfer fluids like water, ethylene glycol, engine oil, etc. have lower heat transfer abilities when compared to solids [1]. Fluids dispersed with nano sized particles (around 10 nm) have shown to improve the thermal conductivity of the fluid [2]. Conventional fluids were replaced by ferrofluids to augment the performance of thermal devices in industries. Over the years, ferrofluid has emerged as a reliable material in solving several complex engineering problems [3]. Ferrofluids have been used in areas such as medicine, transformer cooling, nuclear fusion and chemical engineering [4], [5].

Ferrofluid is a fluid which is prepared by dispersing magnetic nano particles in a carrier liquid. Ferrofluids have the ability to change their physical properties under an externally applied magnetic field and temperature gradient. The magnetisation of the ferrofluid depends on magnetic field, temperature and density. Hence any variations in these quantities give rise to convection (Rayleigh-Bénard convection) in ferrofluids in the presence of magnetic field [6].

Convection in ferrofluids has been a topic of great interest during the last few decades due to its wide range of applications. The mathematical model for ferrofluids was first proposed by Rosensweig and Neuringer [7] where the motion of the fluid in the presence of magnetic field was studied.

Roxanne Francis et al. [8] explored the effects of gravity-modulated Rayleigh-Bénard convection in a Newtonian liquid confined by rigid-free boundaries and other specified conditions. Meenakshi N. and P.G. Siddheshwar [9] delved into Rayleigh Bénard Magnetoconvection within Newtonian Nano liquids, investigating the impact of rotational, gravitational, and temperature modulations. Ansa Mathew and S. Pranesh [10] focused on analysing the onset of Rayleigh-Bénard convection and its heat transfer dynamics under two-frequency rotation modulation. Gravity modulation can induce instabilities that either enhance or suppress convection depending on the modulation frequency and amplitude. Collectively, these studies contribute to advancing the understanding of Rayleigh-Bénard convection across diverse fluid systems and modulation techniques, offering valuable insights into optimizing heat transfer processes essential for various industrial and scientific applications. While extensive research exists on gravity modulation in ferrofluids, the literature regarding gravity modulation in ferrofluids containing nanoparticles is notably absent. Therefore, this paper addresses this gap in research focus.

4. PROBLEM FORMULATION

The physical representation of the problem includes an infinite horizontal layer of Newtonian ferrofluid between the two parallel plates at $z=\pm {}^{d}/_{2}$ with a total width of 'd'. The flow is induced by magnetic field $\vec{H}$ applied normal to the flow. The plates are kept at different, but constant temperature. The fluid layer is affected by time dependent gravitational force $\overrightarrow{g(t)}=-g_0[1+\epsilon f(\gamma,t)]\,\hat{k}$, where $f(\gamma,t)$ is either sinusoidal or non-sinusoidal wave forms.

The conservation equations of mass, linear momentum and energy, the magnetic equation of state and Maxwell's equation are:

$$\nabla\cdot\vec{q}=0 \tag{1}$$

$$\rho_{f_0}=\left(\frac{\partial\vec{q}}{\partial t}+(\vec{q}\cdot\nabla)\vec{q}\right)=-\nabla p+\mu_f\nabla^2\vec{q}+\mu_0(\vec{M}\cdot\nabla)\vec{H}-\left(1+\epsilon f(\gamma,t)\right)\left(\phi_N\rho_p+\rho_{0_f}(1-\phi_N)\left(1-\alpha_t(T-T_0)\right)\right)g_0\hat{k} \tag{2}$$

$$\frac{\partial\phi_N}{\partial t}+(\vec{q}.\nabla)\phi_N=D_B\nabla^2\phi_N+\frac{D_T}{T_0}\nabla^2T-\frac{D_H}{H_0}\nabla^2H \tag{3}$$

$$(\rho_c)_f\left(\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T\right)=K_f\nabla^2T+(\rho_c)_p\left(D_B\nabla\phi_N\cdot\nabla T+\frac{D_T}{T_0}\nabla T\cdot\nabla T-\frac{D_H}{H_0}\nabla T\cdot\nabla H\right) \tag{4}$$

$$\nabla\cdot\vec{B}=0,\ \nabla\times\vec{H}=0 \tag{5}$$

$$\vec{M}=\frac{\vec{H}}{H}\left(M_0+\chi_m(H-H_0)-\chi_T(T-T_0)+K_p\left(\phi_N-\phi_{N_0}\right)\right) \tag{6}$$

Boundary conditions: $T=T_0+\Delta T,\ \phi_N=\phi_{N_0}+\Delta\phi_N$, at $z=-{}^{d}/_{2}$

$T=T_0,\ \phi_N=\phi_{N_0}$, at $z={}^{d}/_{2}$

5. SOLUTION METHODOLOGY

To assess the stability of the system, the critical Rayleigh number is calculated using Venezian approach. We have focused on observing the effect of various parameters $M_1, M_3, M_1', M_4, M_4',$

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$R_n, L_e, \Delta\phi_N, P_r, N_A, N_B, N'_A$ on R_{2c}(correction Rayleigh number). The graphs for the linear case depict a variation of R_{2c} with ω for sinusoidal and non-sinusoidal gravity modulations for different values of the paramaters. Additionally, the heat transfer coefficient is determined by solving the non-autonomous Lorenz equation. In nonlinear case, heat transport is analysed by using average Nusselt number. The graphs for nonlinear case show the variation of average Nusselt number with M_1 for varying values of other parameters for sinusoidal and non-sinusoidal wave types of gravity modulation.

6. SIGNIFICANT CONCLUSIONS

The results obtained in the study are concluded as follows:

1. Square wave form is the most stable while sawtooth waveform is the least stable.
2. The effects for the onset of convection are identical for sawtooth and triangular waveforms.
3. P_r has significant effect only on R_{2c} and does not affect R_0.
4. R_{2c} remains positive for all values of ω and R_{2c} tends to zero as ω increases. Thus, ω affects the stability of the system. By varying the amplitude and frequency of gravity modulation, heat transportation can be regulated.
5. Nanoparticles have the ability to increase the impact of convective heat transfer.
6. Variations in the values of M_4 and R_n influence Rayleigh number, whereas variations in N'_A, L_e and N_B does not influence the Rayleigh number.

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