The study of three component convection in a Viscoelastic fluid with two-frequency time-dependent body force

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1. INTRODUCTION & OBJECTIVE

The triple diffusive convection investigates the diffusion involving three components, heat and two solutes, having distinct molecular diffusivities. Numerous applications can be seen in oceanography and other geophysical and astrophysical contexts. The study of gravity and its modulation play a crucial role in optimizing crystal growth in high-quality materials to enhance heat management in advanced technologies and to improve experiments where gravity can be controlled. Various applications can be seen in impactful fields such as pharmaceuticals, electronics and space research, where the variation of gravity helps in scientific and technological advancements. The time-dependent variation of the gravitational force in a system is called gravity modulation, or g-gitter, that arises from external factors such as oscillations or vibrations. Two-frequency modulation has its advantages over single-frequency modulation, as it increases the robustness and the signal transmission efficiency. Viscoelastic fluids are a non-Newtonian fluid class that unveils dual viscosity and elasticity characteristics. This inimitable behaviour makes them ideal for its application in biomedical devices and industrial processes, which improves the fluids' heat transport efficiency.

While most studies on gravity modulation and its impact on Rayleigh-Bénard convection (RBC) have focused on single-frequency time-periodic modulation. Research on the interaction between triple-diffusive convection and gravity modulation, particularly involving viscoelastic fluids, is relatively scarce. The initial study on the stability of a heated fluid layer from above or below in the presence of a time-dependent buoyancy force was done by Gresho and Sani [1]. Bajaj [2] did the initial study on the two-frequency gravity modulation and discussed how the stability decreased for smaller wave numbers under the influence of modulation. Siddheshwar et al. [3], in their study on the effect on the amount of heat transfer for a time-periodic body force in a viscoelastic fluid, concluded that there was a reduction in the amount of transfer for heat in the case of a time-periodic vertical oscillation. A comparative study on the linear and non-linear three-component convection between sinusoidal and non-sinusoidal gravity modulation was done by Pranesh et al. [4]. Arshika et al. [5] conducted a study on the influence of sinusoidal and non-sinusoidal waveforms on three component convection with time-dependent internal heat source in viscoelastic fluids and compared how the heat source and sink affect the heat and mass transfer for different types of waveforms. Pranesh and Richa [6] investigated the onset of convection and mass and heat transport in triple diffusive convection, in the presence

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of gravity modulation and cross effects in a viscoelastic fluid. Mathew and Pranesh [7] studied the effect of two-frequency rotation modulation on the onset of Rayleigh-Bénard convection and heat transfer and found that the two-frequency modulation was the most stable, reducing the transfer of heat, compared to single frequency modulation and no modulation case. The two-frequency modulation of heat in a Rayleigh-Bénard system with the comparison of sinusoidal waveforms and non-sinusoidal waveforms was investigated by Mathew and Pranesh [8]. A study on the impact of the onset of Rayleigh-B´enard convection with two-frequency temperature modulations at the boundary in a Newtonian fluid was conducted by Mathew and Pranesh [9].

This study presents the first work on two-frequency gravity modulation on triple diffusive convection in Viscoelastic fluids with various combinations of sinusoidal and non-sinusoidal waveforms, for a stress-free isothermal boundary condition for various combinations of sinusoidal and non-sinusoidal waveforms.

Mathematical formulation:

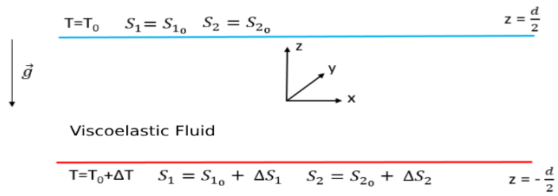

Figure 1: The physical configuration for a three-component convection in a viscoelastic fluid under the influence of two-frequency gravity modulation.

The following governing equations are used to study the problem:

$$\nabla.\vec{q} = 0 \ , \tag{1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p + \rho \vec{g}(t) + \nabla.\tau' \ , \tag{2}$$

$$\left[1 + \lambda_1 \frac{\partial}{\partial t}\right] \tau' = \mu \left[1 + \lambda_2 \frac{\partial}{\partial t}\right] (\nabla \vec{q} \ + \nabla \vec{q}^{Tr}) \ , \tag{3}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)\mathrm{T} = \chi_T \nabla^2 T \ , \tag{4}$$

$$\frac{\partial S_1}{\partial t} + (\vec{q}.\nabla)S_1 = \chi_{S1} \nabla^2 S_1 \ , \tag{5}$$

$$\frac{\partial S_2}{\partial t} + (\vec{q}.\nabla)S_2 = \chi_{S2} \nabla^2 S_2 \ , \tag{6}$$

$$\rho = \rho_0 (1 - \alpha_t (T - T_0) + \alpha_{S1}(S_1 - S_{10}) + \alpha_{S2}(S_2 - S_{20}) \ , \tag{7}$$

where,

$$\vec{g}(t) = -g_0 \left(1 + \varepsilon \ \textstyle\sum_{i=1}^{2} \delta_i f_i \ (t)\right) \hat{k} \ , \tag{8}$$

$$\textstyle\sum_{i=1}^{2} \delta_i f_i \ (t) = \ \delta_1 \ f_1(t) + \delta_2 \ f_2(t) = \delta_1 \ \cos(\chi) \ g_1(r_1 \gamma t) \ + \delta_2 \ \sin(\chi) \ g_2(r_2 \gamma t) \ . \tag{9}$$

The boundary condition for temperature and mass transfer are given by:

$$T = T_0 + \Delta T, S_1 = S_{1_0} + \Delta S_1 \text{ and } S_2 = S_{2_0} + \Delta S_2 \text{ at } z = 0$$
$$T = T_0, S_1 = S_{1_0} \text{ and } S_2 = S_{2_0} \text{ at } z = d \qquad (10)$$

Where d is dimensional layer depth (m), t is time (s), $\vec{q}$ is velocity of the fluid (m s^{-1}), ρ is density of the fluid (kg/m³), ρ_0 is reference density, p is pressure (N/m²), μ is viscosity of the fluid (Pa.s), χ_t is thermal conductivity of the fluid (W/m·K), χ_{S1} is solute 1 diffusivity (m²/s), χ_{S2} is solute 2 diffusivity (m²/s), χ is mixing angle (radians), α_t is coefficient of thermal expansion (K⁻¹), α_{S1} is coefficient of solute 1 expansion (K⁻¹), α_{S2} is coefficient of solute 2 expansion (K⁻¹), $\vec{g}$ is gravitational acceleration (m/s²), T is temperature of the fluid (K), T_0 is reference temperature, S_1 is solute 1 concentration (mol/L), S_2 is solute 2 concentration (mol/L), τ' is stress tensor (N/m²), λ_1 is stress relaxation time (s), λ_2 is strain retardation time (s), δ_1 and δ_2 are amplitudes of modulation (V), γ is frequency of modulation (Hz), r_1 and r_2 are co-prime integers, g_0 is acceleration due to gravity in the absence of modulation(m/s²).

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

By following the usual stability analysis procedure, we eliminate pressure and perform non-dimensionalization to simplify the equations using the stream functions. To further analyze the stability, we follow the Venzian approach and obtain the following expressions for critical parameters, such as R_0, R_1 and R_{2C} . The expressions for R_0, R_1 and R_{2C} obtained are:

$$R_0 = \frac{R_{S1}}{\tau_1} + \frac{R_{S2}}{\tau_2} + \frac{k^6}{\pi^2 \alpha^2} \quad , \qquad (11)$$

$$R_1 = 0 \quad , \qquad (12)$$

$$R_{2C} = \frac{\pi^2 \alpha^2 Z_6{}^2 Y_1((\delta_1 \cos(\chi))^2 + (\delta_1 \sin(\chi))^2)}{2(Y_1{}^2 + Y_2{}^2)\tau_1 \tau_2 k^4}. \qquad (13)$$

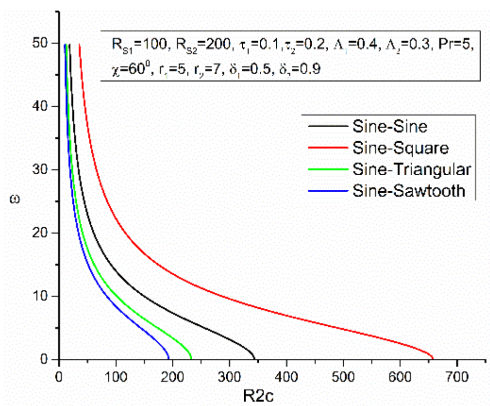

Figure 2: Plot of correction Rayleigh number versus frequency of modulation for different combinations of sine waveform with sinusoidal and non-sinusoidal waveforms.

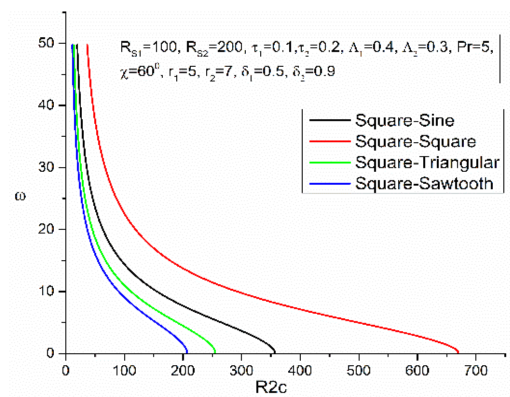

Figure 3: Plot of correction Rayleigh number versus frequency of modulation for different combinations of square waveform with sinusoidal and non-sinusoidal waveforms.

Conclusions:

1. The sine-square waveforms combination is the most stable as compared to other sinusoidal and non-sinusoidal combinations.

2. The square-square waveforms combination delays the onset of convection as compared to other sinusoidal and non-sinusoidal combinations.
3. The combinations of different waveforms can be used to alter the onset of convection.

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