Study of onset of convection in a natural convection for rigid boundaries under the influence of internal heat generation

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1. INTRODUCTION & OBJECTIVE

This paper explores the onset of natural convection in the presence of a uniform internal heat source and rigid boundaries, assuming thermal equilibrium between the fluid and solid phases. Two distinct thermal mechanisms are investigated, with one mechanism dominating the other in each scenario. A novel approach is utilized to conduct a linear stability analysis for both cases. The eigenvalues for the two problems differ: in the first case, the eigenvalue is defined by the Rayleigh number associated with internal heat generation, while in the second, representing classical Bénard convection, it corresponds to the buoyancy-driven Rayleigh number.

Initial estimates of the critical Rayleigh number for both cases are determined using the single-term Galerkin method, and these estimates are further refined using the Maclaurin series method. The paper thoroughly examines the impact of key convection parameters on system stability. Results indicate that higher values of the porous parameter and Brinkman number enhance system stability. The percentage error between eigenvalues derived from the single-term Galerkin method and the Maclaurin series method is also analyzed.

In the second convection scenario, two Rayleigh numbers are considered: the weak internal Rayleigh number, R_I, and the external Rayleigh number, Ra. The influence of R_I on Ra is observed to lower the latter in the presence of a heat source and raise it when a heat sink is involved. Additionally, the conditions facilitating the transition from Brinkman-Bénard to Darcy-Bénard convection are identified and discussed.

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

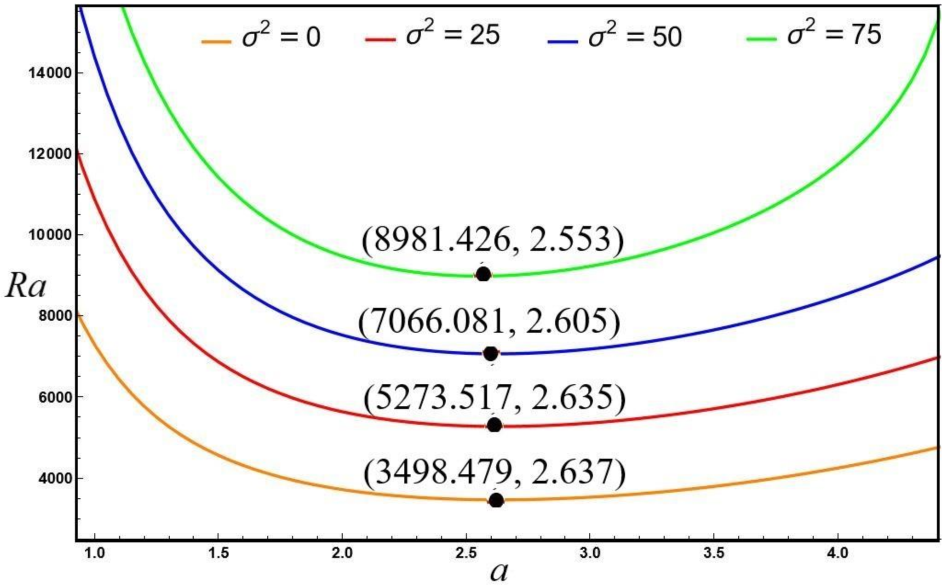
This study focuses on two types of natural convection problems in the presence of internal heat generation. The first problem examines convection driven by heat generation, while the second explores Brinkman-Bénard convection. In the first case, the Rayleigh number, Ra, is the primary driving force, with internal heat generation having only a mild effect on enhancing or suppressing convection. The second problem introduces an internal Rayleigh number, R_I, where the classical Bénard-type buoyancy Rayleigh number, Ra serves as the eigenvalue.

For the first formulation, it is assumed that the internal heat source is weak enough not to independently drive convection, so the heat generation parameter is incorporated into the definition of Ra. In the second formulation, small positive values of RI represent a heat source, while negative values indicate a heat sink.

A linear stability analysis was initially performed using the single-term Galerkin method (STGM) with a trial function, yielding an approximate solution. A more accurate solution was obtained using a novel approach that combines the Maclaurin series and the Newton-Raphson method. To achieve numerical accuracy to three decimal places, 25 terms in the power series expansion were considered for both problems. The STGM was also used to identify the wavenumber range in which the critical value lies, and to provide a good initial estimate for the eigenvalue.

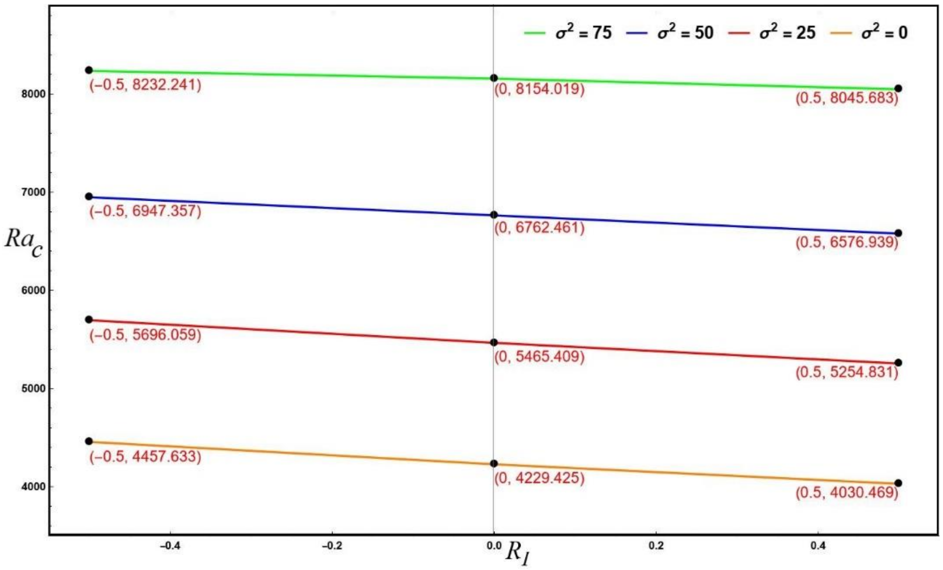
From the first formulation, we obtain Figure 1. which helps us to conclude that, an increase in σ^2, results in reduced permeability of the porous medium, restricting the fluid flow space and hence the , Ra_c, increases

Figure 1. Variation of the critical Rayleigh number, Ra_c , with the porous parameter, σ^2, for a fixed value of the Brinkman number, $\Lambda = 1.25$.



From second formulation, we obtain Figure 2, which helps us conclude that a heat source, represented by positive values of R_I, advances the onset of convection, while a heat sink, represented by negative values of R_I, delays the onset of convection

Figure 2. Variation of the critical Rayleigh number, Ra_c, with internal Rayleigh number, R_I, for a fixed value of $\Lambda = 1.5$



REFERENCES

1. D. A. Nield, A. Bejan, et al., *Convection in porous media*, vol. 3. Springer, USA, 2006.
2. K. Vafai, Handbook of porous media. CRC Press, USA, 2015.
3. P. G. Siddheshwar, P. Stephen Titus, “Nonlinear Rayleigh–Bénard convection with variable heat source,” *ASME J. of Heat Transfer*, **135**, pp. 122-136, 2013
4. P. H. Roberts, “Convection in horizontal layers with internal heat generation. Theory,” *J. Fluid Mechanics*. **30**, pp. 33-49, 1967.