Darcy-Brinkman-Forchheimer flow of a Newtonian fluid and heat transfer through an enclosure with two straight boundaries and one curved boundary.

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1. INTRODUCTION & OBJECTIVE

The study of fluid flow through porous media is a crucial aspect in understanding various natural and industrial processes, such as groundwater flow, enhanced oil recovery, and heat storage systems. The Darcy-Brinkman-Forchheimer model is widely used to describe the flow of a viscous fluid through a porous medium, as it incorporates both the Darcy permeability effect and the inertial forces through the Forchheimer term. However, the nonlinearity introduced by the Forchheimer term, coupled with the complexity of the Brinkman extension, makes it challenging to solve this system analytically, particularly in two- or three-dimensional settings. Adding to this challenge is the presence of irregular or curved boundaries, which are often encountered in practical applications but significantly complicate the computational modeling of such flows. This difficulty motivates the use of numerical methods to approximate solutions. Recent studies have highlighted the effectiveness of central difference approximations and the alternating direction implicit (ADI) scheme in solving complex problems in irregular geometries. By applying quasi-linearization techniques to linearize the nonlinear governing equations and using a grid with unequal mesh points, we can obtain accurate approximations for flow velocity and temperature distribution. These methods, combined with the generalized trapezoidal rule for evaluating integrals like the Nusselt number, are particularly useful in capturing the critical aspects of fluid flow and heat transfer in porous media. Previous works by Hooman and Vafai have demonstrated the importance of such approaches in understanding the effects of porous parameters and Forchheimer numbers on fluid flow in heat transfer applications (Hooman, 2017; Vafai, 2005). Therefore, the current study builds on this foundation, extending it to solve for flow velocity, temperature distribution, and Nusselt number in an enclosure with both straight and curved boundaries.

<u>Objectives</u>

The primary objective of this study is to numerically solve the Darcy-Brinkman-Forchheimer flow of a Newtonian fluid through an enclosure with two straight boundaries and one curved boundary. The specific goals include:

1. Developing a computational method that utilizes the central difference approximation with unequal mesh points to solve for the velocity and temperature profiles.
2. Applying the quasi-linearization technique in conjunction with the ADI scheme to obtain a system of linear algebraic equations from the governing nonlinear partial differential equations.
3. Investigating the influence of porous parameters and Forchheimer numbers on the flow velocity and temperature distribution.

4. Calculating the Nusselt number by evaluating a double integral with variable limits, using the generalized trapezoidal rule, and examining how the porous parameter affects the convective heat transfer rate.

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

The numerical analysis of the Brinkman-Forchheimer equation with a curved boundary reveals that the fluid velocity decreases as the porous parameter increases. This indicates that higher resistance in the porous medium impedes the flow. Similarly, an increase in the Forchheimer number also reduces the fluid velocity, reflecting the impact of increased inertial effects. The temperature distribution shows a comparable trend, decreasing with both the porous parameter and Forchheimer number. This behavior suggests that both the resistance within the porous medium and inertial effects influence thermal transport. The Nusselt number, which measures the convective heat transfer rate, increases with the porous parameter. This result, obtained using a double integral with variable limits and solved numerically by the generalized trapezoidal rule, indicates enhanced heat transfer with a higher porous parameter despite the reduction in fluid velocity. The numerical approach achieved a solution accuracy of 10^{-4}, ensuring reliable and precise results for both velocity and temperature profiles.

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