Effects of Internal Heat Generation on Heat and Mass Transfer in Triple Diffusive Convection under Different Boundary Combinations

Arun Surya E S[a], S Pranesh[b] and Kavya S Kumar[c]

[a]Centre of Mathematical Needs, Christ University, Bangalore, India
[b]Centre of Mathematical Needs, Faculty, Christ University, Bangalore, India
[c]Centre of Mathematical Needs, Christ University, Bangalore, India

1. ABSTRACT

A study of non-linear triple diffusive convection in a Newtonian fluid with internal heat source or sink is investigated in this paper. Sixteen boundary combinations are considered in this paper with symmetric and asymmetric boundary of realistic type. The results of this boundaries are compared with idealistic type. Seven-mode Lorenz model is derived using Fourier expansion to study the non-linear dynamics of the problem. The effect of heat and mass transfer are analyzed by obtaining the expression for time-dependent Nusselt and Shearwood numbers by solving the Lorenz equations numerically. The comparitive study of one,two and three component convection are obtained as a limiting case of the problem. It is found that in three component convection heat and mass transfer is decreased compare to one and two component.

2. INTRODUCTION

Fluids are basically substances that undergo continuous deformations under the action of an external force. In general, liquids and gases come under the category of fluids. The general form of transfer of heat in any substance is conduction, convection and radiation. Convection is the important mode of heat transfer in fluids. Convection is based on the principle of heat transfer by the bulk movement of fluids. It is a form of heat transfer that occurs through the motion of fluids, including liquids, gases, and even the Earth's mantle. Natural convection is a form of heat transfer that occurs spontaneously due to temperature differences within a fluid. It relies on the principle that when a fluid is heated, it expands and becomes less dense, causing it to rise, while cooler fluid sinks to replace it. This process creates a continuous circulation of fluid, known as a convection current, which facilitates the transfer of heat. Density differences within a fluid drive natural convection. Also gravitational effect as well as buoyancy force affect natural convection.

In this studies we focuses on stability analysis in a type of natural convection known as RBC, in the presence of two solute concentrations. The mathematical formulation of our problem along and its physical configuration are described and we obtain non-dimensionalized perturbation equations from the original governing equations. Then we look into the Nonlinear analysis. The non-linear study is important since the linear theory fails to quantify the heat and mass transports. Using higher mode truncated Fourier series representation. we make a non-linear study of the system of Equations subject to the boundary condition. Then the stream function, temperature distribution and solutal concentration distribution are obtained. The main focus of this study is on the investigation of the effects of heat and mass transports in a triple diffusive system, which are quantified by Nusselt number and Sherwood numbers.

3. MATHEMATICAL FORMULATION

The mathematical formulation 0f the problem of Rayleigh-Benard Convection [triple diffusion] include creating the governing equations for the solute concentration, heat transfer, fluid velocity, and related boundary conditions.The specific formulas and configurations are obtained using the conservation laws pertaining to mass, momentum, and energy. Analysis is done after perturbations or disturbances are applied to the basic states.

3.1 Physical configuration

Consider an infinitely extended horizontal layer of Newtonian fluid of uniform vertical thickness d. Let the lower and upper plate be at $z = -\frac{1}{2}$ and $z = \frac{1}{2}$ respectively. Let the layer be heated from below with temperature $T = \Delta T + T_0$. and the difference between temperatures of both the plates are ΔT. The temperature at the upper plate is $T = T_0$. Let ΔC_1 and ΔC_2 be the concentration difference between the two plates situated at $z = \frac{d}{2}$ and $z = -\frac{d}{2}$.

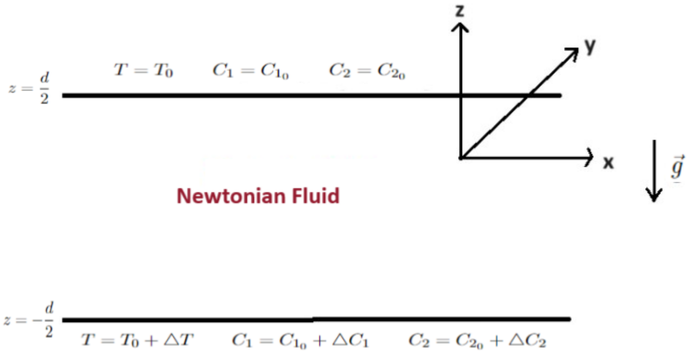

Figure 1: Physical Configuration

3.2 Governing Equations

The following are the governing equations:

- **Equation of Continuity:**

$$\nabla . \vec{q} = 0, \tag{1}$$

- **Conservation of Linear Momentum:**

$$\rho_o \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} \right] = -\nabla p - \rho g \hat{k} + \mu \nabla^2 \vec{q}, \tag{2}$$

- **Conservation of Energy:**

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \chi \nabla^2 T + Q(T - T_o). \tag{3}$$

- **Conservation of soluted concentration 1:**

$$\frac{\partial C_1}{\partial t} + (\vec{q}.\nabla)C_1 = \chi_{s1} \nabla^2 C_1, \tag{4}$$

- **Conservation of soluted concentration 2:**

$$\frac{\partial C_2}{\partial t} + (\vec{q}.\nabla)C_2 = \chi_{s2} \nabla^2 C_2, \tag{5}$$

- **Equation of State:**

$$\rho = \rho_o[1 - \alpha_t(T - T_o) + \alpha_{s1}(C_1 - C_{1o})] + \alpha_{s2}(C_2 - C_{2o})]. \tag{6}$$

4. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

The heat and mass transfer is quantified by Nusselt Number and Shearwood Numbers given by

- **Nusselt Number:**

$$Nu = 1 + (\frac{2\pi tan\sqrt{R_i}}{\sqrt{R_i}})C(t). \tag{7}$$

- **Shearwood Number 1:**

$$Sh_1 = 1 + 2\pi E(t), \tag{8}$$

- **Shearwood Number 2:**

$$Sh_2 = 1 + 2\pi G(t), \tag{9}$$

4.1 Variation of Rs1 in FIFI

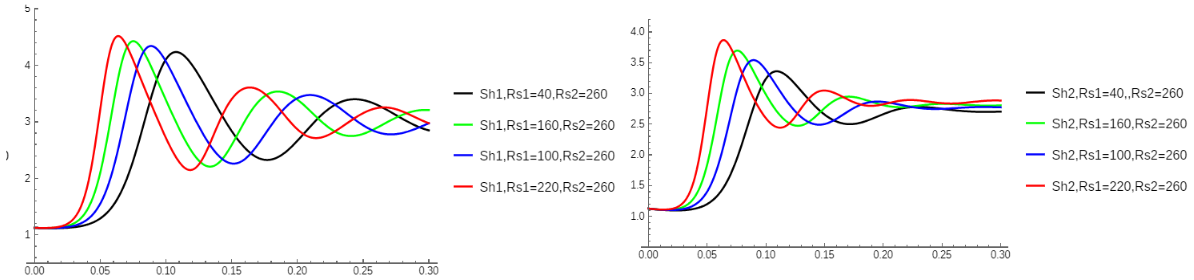

Figure 2: Plot for Sh1 and Sh2 vs t for different values of Rs1

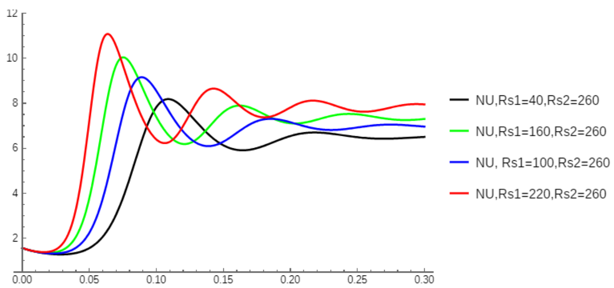

Figure 3: Plot for Nu vs t for different values of Rs1

4.2 Variation of τ_1 in FIFI

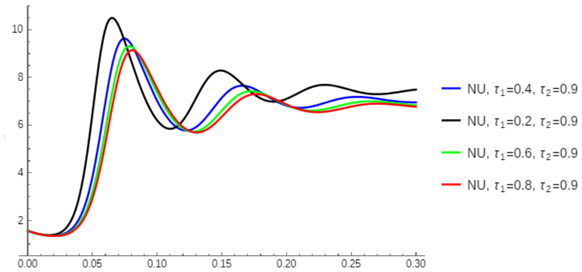

Figure 4: Plot for Nu vs t for different values of τ_1

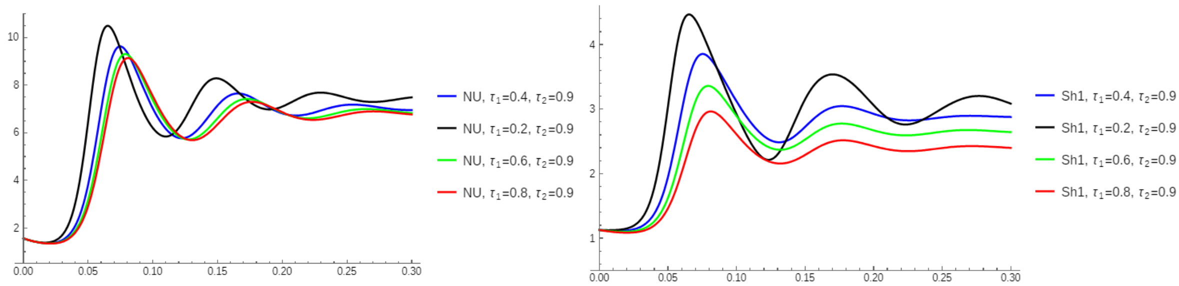

Figure 5: Plot for Sh1 and Sh2 vs t for different values of τ_1

From the above Figures 2 and 3 we get the plot of Nusselt Number, Shearwood numbers vs Time for different values of Rs1 in Free- Free Isothermal. In this case we can say that when Rs1 increases Nu,Sh1,Sh2 increases. So Rs1 increases the heat and mass transfer of the system.

From the Figures 4 and 5 we get the plot of Nusselt Number, Shearwood numbers vs Time for different values of Tau1 in Free- Free Isothermal. In this case we can say that when τ_1 increases Nu,Sh1,Sh2 decreases. So τ_1 decreases the heat and mass transfer of the system.

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